# Committee decisions under biased persuasion*

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**Abstract**

We study information transmission through biased persuasion where an expert whose interests are partially in conflict with a group of committee members releases credible but strategically coarse information in an attempt to manipulate committee decisions. We show that even if members welcome the additional information when it arrives, the expert's presence can hurt their ex-ante welfare in both large and small committees.

*Keywords* Committee decision · Public persuasion · Welfare

**JEL** D60 · D71 · D72 · D82

## 1 Introduction

Employing an expert advisor to provide inputs to a committee is a common practice that is recurrently pursued to abet the process of correct decision-making. It is often the case that owing to concerns regarding her reputation, the expert does not provide information which is incorrect, and this is common knowledge. However, it is entirely possible that the expert may have her personal biases that

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are partially in conflict with the preferences of the committee members. While public knowledge about the expert's reputation concerns makes the expert better placed to credibly transmit any information in spite of her biases, this power of credibility may in turn enhance her ability to manipulate committee decisions. This the expert does by suitably tailoring the content of the advice provided in an effort to manipulate committee actions. It then remains ambiguous what is better for the voters: the availability of credible information (while being exposed to possible manipulation) or not receiving any additional information (and being impervious to any manipulation). Given this ambiguity we ask what are the theoretical consequences of the presence of credible but strategic release of information on the welfare of the committee members. In particular, we ask if the availability of additional public information unambiguously improves equilibrium welfare of the members.

To address this question we extend the Jury model a la Austen-Smith and Banks (1996) (with an odd number $n$ of voters) by including an additional player who does not vote but has free access to information. The voters face uncertainty over the true state of the world and must choose collectively (via the majoritarian voting rule) between two alternatives, $X$ and $Y$. They have common preferences over states and alternatives, hold a common prior and each voter receives private information about the state. The expert has free access to an array of information disseminating technologies that generate informative public signals about the true state in varying degrees of precision and has the ability to commit to any such technology publicly. However, the expert's preferences are not perfectly aligned with those of the voters: while voters prefer $X$ in some states and $Y$ in others, the expert prefers $X$ in all states and this is common knowledge.[1] Once voters receive a public signal that the expert's chosen signal generating technology provides regarding the state of the world, voting takes place.

As the model is akin to a common interest game for the $n$ voters, the celebrated result of McLennan (1998) holds: the 'optimal' strategy profile remains a Nash equilibrium that therefore aggregates information most efficiently so that the efficiency of this equilibrium necessarily gets enhanced, irrespective of the voting rule, whenever there is additional information from an outside expert. But the voting game yields multiple Nash equilibrium and the contribution of our paper is to show that there exists another Nash equilibrium (in spirit of the one studied in Austen-Smith and Banks) such that public information can distort the outcome in favour of the expert and hurt voter welfare. However, the qualification of this result is nuanced and depends upon various aspects of the environment including the number of voters, degree of conflict between the voters and the expert, and

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other informational parameters like priors and signal strengths of private information and we provide a complete characterisation of this interplay. In general, we show that presence of an expert who has free access to technologies that can disseminate information can hurt voter welfare in a Nash equilibrium when $n$ is large and private signals are highly informative. However, our most interesting results are for committees with small $n$ (viz. $n \leq 5$) and in particular when the bias of the expert is small. For that case we show that the welfare effect of persuasion is non-monotonic in the precision of private information.

Austen-Smith and Banks also studied the impact of public information on committee decisions. Although the framework for the dissemination of public information was non-strategic in their framework, they showed that there is no pure strategy Nash equilibrium where each voter voted sincerely and informatively when voters receive two private signals and any number of public signals. Moreover, the welfare implications of that work (for an arbitrary committee size $n$) is limited as the authors are focussed on obtaining informative voting in equilibrium so that the probability of correct decision making increases with $n$. The role of a persuasive expert is studied by Kamenica and Gentzkow (2011) in a framework where the receiver of information is a single decision maker rather than a committee. As in the present paper, they also characterise sender-optimal signals and the possibility that the sender strictly benefits. As noted by the authors, their analysis can be extended to the case of multiple receivers where the expert can persuade by revealing public information. Our paper provides a detailed analysis of such scenarios in the context of majoritarian committees. The social value of public information has been a well addressed subject since the work of Hirshleifer (1971). In a model with strategic complementarity, Morris and Shin (2002) show that public information can hurt social welfare only if agents also have access to independent sources of information. On the other hand, in the investment game of Angeletos and Pavan (2004) public information necessarily improves welfare. Also, Angeletos and Pavan (2007) show how welfare properties of public information depends not only on the form of strategic interaction but also on other external effects that determine the gap between equilibrium and efficient use of public information.[2] However in these papers, public information is non-strategic.

The rest of the paper is organized as follows. In Section 2 we describe the model formally. Section 3 characterises the optimal persuasion strategy followed by the expert and the subsequent voting behaviour it induces. Section 4 deals with welfare implications by comparing the scenarios where expert advice is available versus when it is not. We draw our conclusions in Section 5. The appendix (Section 6) contains the proofs of Lemma 1, and three claims that are used in the proofs of

[2]See also Bikchandani et al. (1992), Cao and Hirshleifer (2000) and Gersbach (2000), among others, for related works on impact of public information on social welfare.

Propositions 1 and 3.

## 2 The Model

An odd number of committee members (also referred to as voters) with common preferences vote over two alternatives. Their preferences over the alternatives depend upon an unknown state of the world and each member receives privately an informative signal about the true state. An expert with preferences different from those of the members has free access to information about the unknown state through the choice of an information disseminating technology to transmit information about it publicly to the committee. In accordance to this technology, the public information transmitted is in the form of declaration of a 'range' of states the true state belongs to. Upon receiving private signals and observing the information revealed through the expert's chosen information disseminating technology, the members vote simultaneously. We model this environment in the following way.

$I = \{1, \cdots, n\}$ is the set of committee members ($n \geq 3$ and odd), $A = \{X, Y\}$ is the set of alternatives and $\Omega = [0, 1]$ is the set of states. The state $\omega \in \Omega$ is a random variable and agents have a common prior given by density $f(\omega)$ where $f$ is non-atomic with distribution function $F(\omega)$. The members have a common preference over $A$ represented by the state-dependent strict preference relation $\succ$ such that for some $0 < \omega_v < 1$, we have $X \succ Y$ if $\omega \leq \omega_v$ and $Y \succ X$ if $\omega > \omega_v$. These preferences of the members are represented by the utility function $u : A \times \Omega \to \mathbb{R}$ such that for $\zeta, \tau \in \mathbb{R}, \zeta < \tau$ we have:

$$u(X, \omega) = \begin{cases} \tau & \text{if } \omega \leq \omega_v \\ \zeta & \text{otherwise} \end{cases}$$

and

$$u(Y, \omega) = \begin{cases} \tau & \text{if } \omega > \omega_v \\ \zeta & \text{otherwise} \end{cases}$$

Each member $i \in I$ receives an i.i.d. private signal $s_i \in \{X, Y\} \equiv S$ whose (common) precision is $p \in (1/2, 1)$, that is, $\mathbb{P}[s_i = X | \omega \leq \omega_v] = \mathbb{P}[s_i = Y | \omega > \omega_v] = p$. Let $s = (s_1, ..., s_n)$ denote a feasible signal profile, $s \in \{X, Y\}^n$.

The expert strictly prefers $X$ over $Y$ in all states. This preference of the expert is represented by the utility function $u_m : A \times \Omega \to \mathbb{R}$ such that for $\zeta_m$, $\tau_m \in \mathbb{R}$ with $\zeta_m < \tau_m$ we have $u_m(X, \omega) = \tau_m$ and $u_m(Y, \omega) = \zeta_m$ for all $\omega \in \Omega$.[3] The case

[3]Our results remain qualitatively intact in a more general environment where in some states the expert prefers alternative $Y$.

$F(\omega_v) > 1/2$ will be referred to as a case of *small conflict* between the members and the expert while *large conflict* will correspond to $F(\omega_v) < 1/2$.

The expert transmits information by announcing an *information dissemination technology (i.d.t)* prior to the realization of the true state which is modeled as an interval partition of the state space $\Omega$ such that an interval included in this partition is revealed to the members if and only if the true state lies in that interval. For $k \geq 1$, let $\Omega^k = \{\Omega_1^k, \cdots, \Omega_k^k\}$ denote a $k$-element interval partition of $\Omega$ announced by the expert. A *public signal* generated by such an i.d.t. is essentially an interval $\Omega_t^k$ of $\Omega$ with conditional density $f(\omega|\Omega_t^k)$ that follows the Bayes rule. Let $\mathcal{M}$ be the space of i.d.t-s comprising all $k$ - element partitions of $\Omega$, $k \geq 1$.

Given an i.d.t. $\Omega^k$, a *voting (pure) strategy* for committee member $i \in I$ is a function $v_i : \Omega^k \times S \to A$ that maps a publicly generated signal $\Omega_t^k \in \Omega^k$ and the private signal $s_i$ to a *vote* $v_i \in A$. Let $\mathcal{V}$ be the set containing all possible voting strategies of a voter. We denote by $v = (v_1, \cdots, v_n) \in A^n$ a *vote profile* and use the shorthand $v(\Omega^k, s)$ to denote $(v_1(\Omega_t^k, s_1), ..., v_n(\Omega_t^k, s_n))$. The *committee decision function* $\delta : A^n \to A$ is *majoritarian* and maps a vote profile $v \in A^n$ to an outcome $\delta(v) \in A$ such that $\delta(v) = X$ if and only if $\#\{i \in I | v_i = X\} \geq \frac{n+1}{2}$.

*Equilibrium*: We focus on symmetric (pure strategies) Perfect Bayesian equilibria of the voting continuation game where each voter follows a 'rational voting strategy', a term coined by Austen-Smith and Banks (1996): he votes in favor of the alternative that maximizes his expected utility after having made full use of his available information (which consists of the public signal drawn by the i.d.t. chosen by the expert, the private signal received and inference about the signals of the other members from the pivotal vote profile). We call such an equilibrium the *rational voting equilibrium* (RVE). Given an i.d.t $\Omega^k$, a voting strategy profile $v = (v_1(\Omega_t^k, s_1), ..., v_n(\Omega_t^k, s_n))$ constitutes an RVE of the voting continuation game if: for each $s_i \in \{X, Y\}$, $i \in I$, and $v_i' \in \mathcal{V}$, we have

$$\sum_{s_{-i} \in \{X,Y\}^{n-1}} \left( \int_{\omega \in \Omega_t^k} \mathbb{P}[s_{-i}|\Omega_t^k, Piv_i] u(\delta(v(\Omega_t^k, s_i)), \omega) f(\omega|s) d\omega \right) \geq$$
$$\sum_{s_{-i} \in \{X,Y\}^{n-1}} \left( \int_{\omega \in \Omega_t^k} \mathbb{P}[s_{-i}|\Omega_t^k, Piv_i] u(\delta(v_i'(\Omega_t^k, s_i), v_{-i}), \omega) f(\omega|s) d\omega \right),$$

where $Piv_i$ is the event that voter $i$ is pivotal. We note here that for every i.d.t. $\Omega^k$ and for each $\omega \in \Omega$, there is always a unique RVE in the voting continuation game. Given this, we proceed to define the equilibrium of the full game. As in

Kamenica and Gentzkow (2011), an i.d.t constitutes an equilibrium of the full game if and only if it maximizes the expert's ex-ante payoffs. Let $(\Omega^k, v)$ such that $v$ is the RVE of the continuation voting game given the i.d.t. $\Omega^k$. Then $(\Omega^k, v)$ is an equilibrium of the full game if for all other pairs $(\Omega^{k'}, v')$ such that $v'$ is the RVE of the continuation voting game given the i.d.t. $\Omega^{k'}$, we have

$$\int_{\omega\in\Omega}\left(\sum_{s\in\{X,Y\}^n}\mathbb{P}[s|\omega]u_m(\delta(v(\Omega^k,s)),\omega)\right)f(\omega)d\omega$$

$$\geq \int_{\omega\in\Omega}\left(\sum_{s\in\{X,Y\}^n}\mathbb{P}[s|\omega]u_m(\delta(v'(\Omega^{k'},s)),\omega)\right)f(\omega)d\omega.$$

There can be multiple equilibria of the full game, but since all equilibria will be payoff-equivalent for the expert (and hence payoff equivalent for the voters), we shall consider the coarsest equilibrium i.d.t.s in the rest of the paper.

We identify committee welfare in terms of the *ex-ante welfare* of the committee members prior to any non-prior information received. Note that this is fully explained by any individual member's cardinal preferences since all members are ex-ante identical. So let $U(\Omega^k, v)$ be the *ex-ante welfare of a representative voter* under a strategy profile $(\Omega^k, v)$. Then

$$U(\Omega^k,v) = \int_{\omega\in\Omega}\left(\sum_{s\in\{X,Y\}^n}\mathbb{P}[s|\omega]u(\delta(v(\Omega^k,s)),\omega)\right)f(\omega)d\omega.$$

# 3 Equilibrium persuasion

To fix our benchmark, we first characterise the rational voting equilibrium in the absence of persuasion.

**Lemma 1.** *Without public persuasion, the unique RVE is as follows: for each voter $i = 1, ..., n$,*

*(a) if $F(w_v) > 1/2$ then $v_i(s_i) = s_i$ if $p > F(\omega_v)$ and $v_i(s_i) = X$ for each $s_i \in S$ if $p < F(\omega_v)$;*

*(b) if $F(w_v) < 1/2$ then, $v_i(s_i) = s_i$ if $p > 1 - F(\omega_v)$ and $v_i(s_i) = Y$ for each $s_i \in S$ if $p < 1 - F(\omega_v)$.*

It is important in this stage to note that in only two scenarios (viz. $p > F(\omega_v) > 1/2$ and $1/2 > F(\omega_v) > 1 - p$) does one obtain an RVE in our model without public persuasion where individual votes are exactly in accordance with individual signals (called *informative* voting in Austen-Smith and Banks (1996)). It is in only these two cases that the celebrated Condorcet Jury Theorem holds so that as $n$ approaches $+\infty$, the probability of correct committee decision approaches 1. As we shall see in what follows, public persuasion will have a qualified power to distort aggregate decisions in these parametric zones of the model as well.

## 3.1 Small bias: $F(\omega_v) > 1/2$

We begin with the case when the conflict of preference between the voters and the expert is small. The characterisation of the equilibrium in the full game in this case is given by the following proposition.

**Proposition 1.** *Let $(\Omega^k, v)$ be an equilibrium of the voting game under biased persuasion and let $F(\omega_v) > 1/2$. Then $k = 2$ and there exists a unique $\omega^* > \omega_v$ such that $\Omega_1^2 = [0, \omega^*]$ and $\Omega_2^2 = (\omega^*, 1]$. Moreover,*

*(a) if $p < F(\omega_v)$, then $\omega^* = 1$ and $v_i(\Omega_1^1, s_i) = X$ for each $s_i \in S$, i.e., persuasion yields no information, and*

*(b) if $p > F(\omega_v)$, then $F(\omega^*) = \frac{F(\omega_v)}{p}$ (that is, $\omega^* > \omega_v$), $v_i(\Omega_1^2, s_i) = X$ for each $s_i \in S$ and $v_i(\Omega_2, s_i) = Y$ for each $s_i \in S$.*

*Proof.* Consider $1/2 < p < F(\omega_v)$. From Lemma 1 it follows that when the chosen i.d.t is $\Omega^1$, then $v_i(\Omega_1^1, s_i) = X$. Hence there does not exist any profitable deviation for the expert from $\omega^* = 1$. This proves part (a) of the proposition.

Suppose $1/2 < F(\omega_v) < p$. Consider the specific class of persuasion strategy $\Omega^2$, which we classify as *Type 1*, in which $\Omega_1^2 = [0, \omega']$, $\Omega_2^2 = (\omega', 1]$, where $\omega_v < \omega'$ such that $v_i(\Omega_1^2, s_i) = X$ and $v_i(\Omega_2^2, s_i) = Y$ for each $s_i \in S$.
Suppose the public signal generated is $\Omega_1^2$. In this case voter $i$ considers the following posterior probability given by

$$\gamma'_{s_i} = \mathbb{P}[\omega \le \omega_v | s_i, \Omega_1^2, Piv_i] = \frac{G}{H},$$

where

$$G = \mathbb{P}[s_i, \Omega_1^2, Piv_i | \omega \le \omega_v, ]\mathbb{P}[\omega \le \omega_v]$$

and

$$H = \mathbb{P}[s_i, \Omega_1^2, Piv_i | \omega \le \omega_v]\mathbb{P}[\omega \le \omega_v] + \mathbb{P}[s_i, \Omega_1^2, Piv_i | \omega_v < \omega \le \omega']\mathbb{P}[\omega_v < \omega \le \omega']$$

Voter $i$ votes $v_i(\Omega_1^2, s_i) = X$ iff $\gamma'_{s_i} > 1/2$, and $v_i(\Omega_1^2, s_i) = Y$ otherwise. We have,

$$\gamma'_X = \frac{pF(\omega_v)}{pF(\omega_v) + (1-p)(F(\omega') - F(\omega_v))}$$

and

$$\gamma'_Y = \frac{(1-p)F(\omega_v)}{(1-p)F(\omega_v) + p(F(\omega') - F(\omega_v))}.$$

Note that $\gamma'_Y = 1/2$ when $F(\omega') = \frac{F(\omega_v)}{p}$. Since $\frac{\partial \gamma'_Y}{\partial F(\omega')} < 0$, it follows that when $F(\omega')$ is greater (less) than $\frac{F(\omega_v)}{p}$, then $\gamma'_Y$ is less (greater) than 1/2. Since $p > 1/2$, we have $\gamma'_{s_i=X} > \gamma'_{s_i=Y}$. Therefore when $\Omega_1^2 = [0, \omega']$ where $F(\omega') = \frac{F(\omega_v)}{p}$ is satisfied, the voter votes $v_i = X$ for each $s_i \in S$ in equilibrium. For the case when $\Omega_2^2 = (\omega', 1]$, it follows that $\mathbb{P}[\omega \leq \omega_v | \Omega_2^2, s_i] = 0$ for each $s_i \in S$, and the voter $i$ votes $v_i = Y$ for each $s_i \in S$ in equilibrium.

Note that in this case the ex-ante payoff of the expert under *Type 1* persuasion strategy is given by

$$\mathbb{E}[u_m]_1 = F(\omega')(\tau_m - \zeta_m) + \zeta_m$$

Since $\tau_m > \zeta_m$, it follows that $\frac{\partial \mathbb{E}[u_m]_1}{\partial \omega'} > 0$, which implies that by choosing $\omega'$ that satisfies the condition $F(\omega') = \frac{F(\omega_v)}{p}$, the ex-ante payoff of the expert is maximized under *Type 1* persuasion strategies. Hence the optimum value of ex-ante payoff of the expert under Type 1 persuasion strategies is given by

$$\mathbb{E}[u_m]_1^* = \frac{F(\omega_v)}{p}(\tau_m - \zeta_m) + \zeta_m \tag{1}$$

Consider the following class of persuasion strategies, classified as *Type 2*: Suppose there exists $\alpha \in [0, \omega_v)$, $\beta \in (\omega_v, 1]$ such that the persuasion strategy $\Omega^3$ chosen by the expert is given by $\Omega_1^3 = [0, \alpha)$, $\Omega_2^3 = [\alpha, \beta)$, $\Omega_3^3 = [\beta, 1]$, where the equilibrium voting strategy followed is:

$$v_i = \begin{cases} X & \text{if } \Omega_1^3 = [0, \alpha) \\ s_i & \text{if } \Omega_2^3 = [\alpha, \beta] \\ Y & \text{otherwise} \end{cases}$$

Note that when a public signal of $\Omega_2^3 = (\alpha, \beta]$ is received, then for $v_i = s_i$ for each $s_i \in S$ to hold in equilibrium, the conditions

$$\hat{\gamma}_X = \mathbb{P}[\omega \leq \omega_v | s_i = X, \Omega_2^3 = (\alpha, \beta], Piv_i] > 1/2$$

and

$$\hat{\gamma}_Y = \mathbb{P}[\omega \leq \omega_v | s_i = Y, \Omega_2^3 = (\alpha, \beta], Piv_i] < \frac{1}{2}$$

need to be satisfied simultaneously. The first inequality reduces to

$$\frac{p}{1-p}(F(\omega_v) - F(\alpha)) + F(\omega_v) > F(\beta)$$

while the second becomes

$$\frac{1-p}{p}(F(\omega_v) - F(\alpha)) + F(\omega_v) < F(\beta).$$

Under the *Type 2* class of persuasion strategy the ex-ante payoff of the expert is denoted by

$$\mathbb{E}[u_m]_2 = (F(\alpha)+F(\beta))(1-J(n,p))(\tau_m-\zeta_m)+F(\omega_v)(1-2J(n,p))(\zeta_m-\tau_m)+\zeta_m \quad (2)$$

where $J(n,p) = \sum_{j=\frac{n+1}{2}}^{n} \binom{n}{j}(p)^j(1-p)^{n-j}$.

Since $\tau_m > \zeta_m$ and $0 < J(n,p) < 1$, this implies that in order to maximize the ex-ante payoff of the expert, the following optimisation problem needs to be solved, which we denote as $(*)$:

Maximise $(F(\alpha) + F(\beta))$ subject to: (i) $\frac{p}{1-p}(F(\omega_v) - F(\alpha)) + F(\omega_v) > F(\beta)$, (ii) $\frac{1-p}{p}(F(\omega_v) - F(\alpha)) + F(\omega_v) < F(\beta)$, (iii) $0 \geq F(\alpha) < F(\omega_v)$, and (iv) $F(\omega_v) < F(\beta) \leq 1$.

Setting $F(\alpha) = 0$ and considering equality in constraint (i), we have $F(\beta) = \frac{F(\omega_v)}{1-p}$. Note that since $p > 1/2$ and $F(\omega_v) > 1/2$, therefore the condition $1 \geq \frac{F(\omega_v)}{1-p}$ can never hold. Now consider $1 < \frac{F(\omega_v)}{1-p}$. In this case the optimum value is given by the relation $F(\beta^*) = 1$, which implies $\beta^* = 1$. Putting the optimum value of $F(\beta^*)$ in constraint (i), we have $F(\alpha^*) = \frac{F(\omega_v)}{p} - \frac{1-p}{p}$. Replacing the optimum values of $F(\alpha^*)$ and $F(\beta^*)$ in equation (2), we have the maximum ex-ante payoff of the expert under equilibria belonging to Type 2 to be

$$\mathbb{E}[u_m]_2^* = \left(\frac{F(\omega_v)}{p} - \frac{1-p}{p} + 1\right)(1-J(n,p))(\tau_m-\zeta_m)+F(\omega_v)(1-2J(n,p))(\zeta_m-\tau_m)+\zeta_m \quad (3)$$

From (1) and (3) it follows that

$$\mathbb{E}[u_m]_2^* - \mathbb{E}[u_m]_1^* = \frac{V}{p} \tag{4}$$

where $V = (\tau_m - \zeta_m)(J(n,p)(F(\omega_v) - 1)(2p-1) + p(2 - F(\omega_v)) - 1)$.
Let $K(n,p,F(\omega_v)) = J(n,p)(F(\omega_v)-1)(2p-1)+p(2-F(\omega_v))-1$. Note that since $F(\omega_v) < 1$ and $p > 1/2$, we have $\frac{\partial K(n,p,F(\omega_v))}{\partial J(n,p)} < 0$. Here we make the following claim, the proof of which is provided in the Appendix.

**Claim 1.** *$J(n,p)$ is increasing in $n$.*

From Claim 1 it follows that if $K(n,p,F(\omega_v)) < 0$ can be shown to hold for $n = 3$, it will hold for $n > 3$. When $n = 3$, we have $J(3,p) = 3p^2(1-p) + p^3$. Hence we have

$$K(3,p,F(\omega_v)) = 4p^4(1-F(w_p))+8p^3(F(\omega_v)-1)+3p^2(1-F(w_p))+p(2-F(\omega_v))-1$$

Note that $\frac{\partial K(3,p,F(\omega_v))}{\partial F(\omega_v)} = -p(4p^3-8p^2+3p+1)$. Also note that $\frac{\partial K(3,p,F(\omega_v))}{\partial F(\omega_v)}|_{p=1/2} = -1/2$, $\frac{\partial K(3,p,F(\omega_v))}{\partial F(\omega_v)}|_{p=1} = 0$, and $\frac{\partial^2 K(3,p,F(\omega_v))}{\partial p^2} = 0$ has no solution in $p \in (1/2,1)$. Hence $\frac{\partial K(3,p,F(\omega_v))}{\partial F(\omega_v)} < 0$ for all $p \in (1/2,1)$. Therefore the maximum value of $K(3,p,F(\omega_v))$ is attained at $K(3,p,1/2) = 2p^4 - 4p^3 + \frac{3p^2}{2} + \frac{3p}{2} - 1$. Now $K(3,p,1/2)|_{p=1/2} = -\frac{1}{4}$, $K(3,p,1/2)|_{p=1} = 0$, and $\frac{\partial K(3,p,1/2)}{\partial p} = 0$ has no solution in $p \in (1/2,1)$. Hence $K(3,p,F(\omega_v)) < 0$ for all $p \in (1/2,1)$, $F(\omega_v) \in (1/2,1)$. This proves that $\mathbb{E}[u_m]_2^* - \mathbb{E}[u_m]_1^* < 0$ holds for all $n \geq 3$.

Note that in an equilibrium under Type 1 persuasion strategies, $\mathbb{E}[u_m]$ is increasing in $\omega'$, and the maximum value of $\omega'$ under this class is attained when the condition $F(\omega') = \frac{F(\omega_v)}{p}$ is satisfied. When $F(\omega') > \frac{F(\omega_v)}{p}$ then $\hat{\gamma_Y} < 1/2$ and we revert to equilibria under Type 2 persuasion strategies with $\alpha = 0$, $\beta = \omega'$ which has been proved to have a lower expected payoff for the expert than the most influential Type 1 equilibrium.[4] This proves part (b) of the proposition and concludes the proof.

□

To understand the proposition, we note that Type 1 and Type 2 classes of persuasion strategies defined in the above proof are the only candidates for coarsest

[4]There is only one other possible class of i.d.t given by $\Omega^2$ that is to be considered where $\Omega_1^2 = [0,\omega')$, $\Omega_2^2 = [\omega',1]$, with $\omega' < \omega_v$ such that $v_i(\Omega_1^2, s_i) = X$ and $v_i(\Omega_2^2, s_i) = Y$ for each $s_i \in S$. This class clearly corresponds to a lower level of exante welfare for the expert than Type 1 class of i.d.t-s, and hence can never constitute a most influential persuasion strategy for the expert.

equilibrium i.d.t.s. The proof shows that the coarsest equilibrium i.d.t is unique and given by the optimal *Type 1* persuasion strategy. When $p < F(\omega_v)$ it follows from Lemma 1 that when no information is provided, every member votes for $X$ irrespective of his private signal if the strength of his private signal is sufficiently low ($p < F(\omega_v)$). Since the expert prefers the alternative $X$ for all states of the world, this is the ideal scenario for her and therefore she chooses an i.d.t that does not transmit any information in equilibrium. This explains part (a) of Proposition 1. However, if the signal strength of the voters is high ($p > F(\omega_v)$), Lemma 1 shows that under no additional information the voters choose according to their private signals, which prompts the expert to intervene in this scenario. The equilibrium persuasion strategy is such that when $\Omega_1^2$ is declared, the intensity of the endorsement is sufficiently strong to ensure that the voters choose $X$ irrespective of their private signals. However, when $\Omega_2^2$ is declared, the lesser preferred alternative of the expert $Y$ is always chosen. Under the most persuasive strategy, the length of the $\Omega_1^2$ interval is maximized. To see this, observe that in order to make the voters adopt a pooling strategy of voting $X$, the advice should provide credible information that is sufficiently strong in favor of $X$ (which means the mass of states greater than $\omega_v$ that may have generated the same signal given the i.d.t needs to be sufficiently less) so that the voters choose $X$ even when he receives a private signal of $Y$. To achieve this end, the i.d.t must be such that all states in $[0, \omega_v]$ (which ensures maximal evidence in favor of states for which the voters favor $X$) should be included in $\Omega_1^2$ along with other states in $(\omega_v, 1]$ . After having included the entire support $[0, \omega_v]$, the right-most point till which $\omega^*$ can be extended while sustaining a resultant pooling strategy of voting $X$ must satisfy the condition $F(\omega^*) = \frac{F(\omega_v)}{p}$. We note here that the expert can do something more simple with an equivalent effect. She can send out the message to support its least preferred alternative but only some of the time. The rest of the time, she can remain silent and the voters will infer that with a decently high probability, the state is the one preferred by the expert. A similar persuasion strategy explained in Kamenika and Gentzkow (2011) for a single decision maker. It is equivalent in terms of impact but relies on less need for direct communication and allows the expert to transmit only, at most, the statecontingent policy endorsement.

**Remark 1.** *The cut-off state $\omega^*$ that describes uniquely the equilibrium persuasion strategy is a decreasing function of $p$. This implies that more informed voters receive more precise public information regarding states in which there is no conflict between them and the expert.*

## 3.2 Large bias: $F(\omega_v) < 1/2$

We now consider the scenario where the degree of conflict between the expert and the committee members is large. For this case, the choice of the equilibrium persuasion strategy depends crucially on what we call Condition ($*$) given below:
**Condition** ($*$):

$$J(n,p) \geq \frac{p(2-F(\omega_v))-1}{(2p-1)(1-F(\omega_v))}$$

The characterisation of the equilibrium persuasion strategy for the expert and the resultant RVE actions of the voters are given by the following proposition.

**Proposition 2.** *Let* $(\Omega^k, v)$ *be an equilibrium of the voting game under biased persuasion and let* $F(\omega_v) < 1/2$*. Then,* $k = 2$ *and there exists a unique* $\omega^* \in (0,1)$ *such that* $\Omega_1^2 = [0, \omega^*]$ *and* $\Omega_2^2 = (\omega^*, 1]$*. Moreover,*

*(a) If* $p > 1 - F(\omega_v)$*, then*

- *(i) If* ($*$) *holds then* $\omega^*$ *satisfies* $F(\omega^*) = F(\omega_v)/p$*, i.e.* $\omega^* > \omega_v$*, with* $v_i(\Omega_1^2, s_i) = X$ *for each* $s_i \in S$ *and* $v_i(\Omega_2^2, s_i) = Y$ *for each* $s_i \in S$*;*
- *(ii) If* ($*$) *does not hold then* $\omega^*$ *satisfies* $F(\omega^*) = (F(\omega_v)/p) - ((1-p)/p)$*, i.e.* $\omega^* < \omega_v$*, with* $v_i(\Omega_1^2, s_i) = X$ *for each* $s_i \in S$ *and* $v_i(\Omega_2^2, s_i) = s_i$*;*

*(b) If* $p < 1 - F(\omega_v)$*, then*

- *(i) for* $n \geq 5$*, the equilibrium i.d.t and voting behavior is same as in (a.i).*
- *(ii) for* $n = 3$*, there exists* $1/2 < p' < 1$ *such that for all* $p < p'$*, the equilibrium i.d.t and voting behavior is same as in (a.i). However, when* $p > p'$*, then* $F(\omega^*) = F(\omega_v)/(1-p)$*, i.e.* $\omega^* > \omega_v$*, with* $v_i(\Omega_1^2, s_i) = s_i$ *and* $v_i(\Omega_2^2, s_i) = Y$*.*

*Proof.* Consider Class 1 and Class 2 type of persuasion strategies as defined in the proof of Proposition 1. Analogous to the proof of Proposition 1, it follows that the ex-ante payoff of the expert in the equilibrium under Class 1 persuasion strategies is given by (1). Analogous to the same proof it also follows that the i.d.t which maximizes the ex-ante payoff of the expert is obtained by solving the optimization problem ($*$).
Now consider $F(\omega_v) < 1/2$, and $p > 1 - F(\omega_v)$, which implies $1 < \frac{F(\omega_v)}{1-p}$. In this case the solution of ($*$) is given by $(\alpha^*, \beta^*)$ such that $F(\beta^*) = 1$ and $F(\alpha^*) = \frac{F(\omega_v)}{p} - \frac{1-p}{p}$. Replacing the optimum values of $\alpha^*$ and $\beta^*$ in (2), we have the maximum ex-ante payoff of the expert under Class 2 persuasion strategies to be given by (3). Parts (a.i) and (a.ii) of this proposition therefore follows from (4).
Now consider $F(\omega_v) < 1/2$, and $p < 1 - F(\omega_v)$, which implies $\frac{F(\omega_v)}{1-p} < 1$. In this

case the solution of $(*)$ is given by $(\alpha^*,\beta^*)$ such that $F(\alpha^*) = 0$, and $F(\beta^*) = \frac{F(\omega_v)}{1-p}$. Replacing the optimum values of $\alpha^*$ and $\beta^*$ in (2) , we have the maximum ex-ante payoff of the expert under equilibria belonging to Class 2 to be

$$\mathbb{E}[u_m]_2^{**} = \left(\frac{F(\omega_v)}{1-p}\right)(1-J(n,p))(\tau_m-\zeta_m)+F(\omega_v)(1-2J(n,p))(\zeta_m-\tau_m)+\zeta_m \quad (5)$$

From (1) and (5) it follows that

$$\mathbb{E}[u_m]_1^{**} - \mathbb{E}[u_m]_2^{**} = \frac{D}{p(1-p)}$$

where

$$D := D(\omega_v, n, p) = F(\omega_v)(\tau_m - \zeta_m)(J(n,p)p(2p-1) - p^2 - p + 1).$$

Let $\eta(n,p) = J(n,p)p(2p-1) - p^2 - p + 1$. Since $p > 1/2$, therefore $\eta(n,p)$ is increasing in $J(n,p)$, which by Claim 1 is increasing in $n$. Note that $\eta(5, 1/2) = 1/4$, $\eta(5,1) = 0$, and $\eta(5,p) = 0$ does not have a solution in $p \in (1/2, 1)$. Hence $\eta(5,p) > 0$ for all $p \in (1/2, 1)$, which implies $D(\omega_v, 5, p) > 0$ for all $p \in (1/2, 1)$. Since $D(\omega_v, 5, p)$ is increasing in $n$, this proves part (b.i).
To prove part (b.ii), consider $n = 3$. Note that $\eta(3, 1/2) = 1/4$, $\eta(3,1) = 0$, and $\eta(3,p) = 0$ has a unique solution in $p \in (1/2, 1)$ given by $p' = \frac{(27-3\sqrt{78})^{\frac{1}{3}}}{6} + \frac{(3\sqrt{78}+27)^{\frac{1}{3}}}{6} \approx .76$. This shows that for all $p \in (1/2, p')$, we have $D(\omega_v, 3, p) > 0$ while for all $p \in (p', 1)$, we have $D(\omega_v, 3, p) < 0$. This proves part (b.ii) and completes the proof. □

The intuitions regarding the design of the equilibrium persuasion strategy for the large conflict case closely follows that of the small conflict one. However, in some specific instances the two cases diverge, where it becomes optimal for the expert to choose persuasion strategies which allow the voters to vote according to their private signals for certain sections of the state space. This is because it is not possible for the expert to make the voters play a pooling strategy of voting $X$ for the majority of the states, since the prior is in favour of the alternative $Y$ owing to the large degree of conflict.

Note that the expression $J(n,p)$ is the probability that a committee of size $n$ and awareness $p$ makes a correct decision when members vote in accordance with their private signals. Then, $(*)$ provides a lower bound on this probability. This lower bound increases in $p$ and decreases in $\omega_v$.

While $(*)$ holds unambiguously when $F(\omega_v) > 1/2$, it is neither universal nor empty when $F(\omega_v) < 1/2$. For this latter case, we provide some sufficiency conditions for $(*)$ to hold. Let

$$Q(n,p;\omega_v) = J(n,p)(F(\omega_v)-1)(2p-1) + p(2-F(\omega_v)),$$

and note that $(*)$ holds if and only if $Q(n,p;\omega_v)-1 \leq 0$.

**High precision of private information**: If $p=1$, then $J(n,p)=1$. Hence in this case, $Q(n,1;\omega_v)-1=0$. Also note that $\frac{\partial(Q(n,1;\omega_v)-1)}{\partial p}|_{p=1} = F(\omega_v) > 0$. This shows that when $p \to 1$, the expression $(Q(n,1;\omega_v)-1) < 0$. Hence $(*)$ is always satisfied for all $n$ if the precision of the signal received individually by the members is high enough.

**Large committee**: Since $J(n,p) \to 1$ as $n \to \infty$ in the limit we have $Q(n,p;\omega_v) - 1 = p(F(\omega_v)-1) < 0$ for all $p \in (1/2,1)$. This shows that condition $(*)$ is always satisfied if the number of members is sufficiently large.

**Intermediate precision of private information, not-too-large bias, small committee**: Moreover, note that $Q(n,1;\omega_v)-1 \leq 0$ if $J(n,p) \geq (p(2-F(\omega_v))-1)/((2p-1)(1-F(\omega_v)))$, which is always satisfied if the RHS of the inequality is less than or equal to 1/2. This yields $p \leq \frac{1+F(\omega_v)}{2}$. Since $(*)$ is valid for the case $p > 1-F(\omega_v)$, we must therefore ensure $1-F(\omega_v) < \frac{1+F(\omega_v)}{2}$, which yields $\frac{1}{3} < F(\omega_v)$. Hence we know that if $\frac{1}{3} < F(\omega_v) < \frac{1}{2}$ and $1-F(\omega_v) < p \leq (1+F(\omega_v))/2$, condition $(*)$ always holds. As a specific example, consider $F(\omega_v) = .35$. In this case $1-F(\omega_v) = .65$, and $\frac{(1+F(\omega_v))}{2} = .675$. Consider $p = .66$, $n=3$. In this case $Q(n,p;\omega_v)-1 \approx -.063 < 0$, and hence $(*)$ is satisfied.

**When $(*)$ does not hold**: The complement of $(*)$ is non-empty as well. We construct an example. Let $n=3$ and $F(\omega_v) = .18$. In this case for intermediate values of the precision of the private signal (that is when $0.63 < p < 0.9$), $(*)$ is violated while for the cases $1/2 < p < 0.63$ or $0.9 < p < 1$, $(*)$ is satisfied. As a specific example, consider $n=3$ and $p=.7$. For these values, $Q(n,p;\omega_v)-1 \approx .017 > 0$, and hence $(*)$ is violated in this case.

# 4 Welfare Analysis

We now compare the ex-ante welfare of the committee members under persuasion with that in the benchmark case where there is no persuasion.

## 4.1 Welfare with small bias: $F(\omega_v) > 1/2$

While with low signal precision the welfare of the members with or without public persuasion is the same, when the signal precision is high, the welfares are different and this difference is interestingly dependent on the size $n$ of the committee. When $n \geq 7$, it is unambiguously higher without persuasion but the analysis gets nuanced for $n \leq 5$ in many novel ways that depend upon the relation between precision of private information with the given preference bias of the expert. We show that for an intermediate range of signal precision, welfare is higher with persuasion, while for extreme values (either high or low) of signal precision it is higher without persuasion. These observations are made precise in the following Proposition.

**Proposition 3.** *Public persuasion with small bias has the following welfare consequences:*

*(a) When $p < F(\omega_v)$, the ex-ante committee welfare is invariant to the presence of persuasion.*

*(b) When $F(\omega_v) < p$,*

*(i) If $n \geq 7$, the ex-ante committee welfare is higher in the absence of persuasion;*

*(ii) If $n < 7$, there exists $1/2 < k^*(n) < p$ such that (1) when $k^*(n) < F(\omega_v) < p$, the ex-ante committee welfare is higher in the absence of persuasion; (2) when $1/2 < F(\omega_v) < k^*(n)$, there exists $F(\omega_v) < \hat{p}(n, F(\omega_v)) < \tilde{p}(n, F(\omega_v)) < 1$ such that for $p \in (F(\omega_v), \hat{p}(n, F(\omega_v))) \cup (\tilde{p}(n, F(\omega_v)), 1)$, the ex-ante committee welfare is higher in the absence of persuasion. However, when $p \in (\hat{p}(n, F(\omega_v)), \tilde{p}(n, F(\omega_v)))$, the ex-ante committee welfare is higher under persuasion.*

*Proof.* Consider $1/2 < p < F(\omega_v)$. Part (a) follows immediately from Lemma 1 and Proposition 1 part (a). Now consider $1/2 < F(\omega_v) < p$. From Lemma 1 it follows that in the absence of an expert the ex-ante voter welfare is given by

$$U(\emptyset, v) = J(n, p)\tau + (1 - J(n, p))\zeta$$

From Proposition 1 it follows that in the presence of an expert, the most influential equilibrium $(\Omega^2, v)$ results in the ex-ante voter welfare being given by

$$U(\Omega^2, v) = \tau F(\omega_v) + \zeta \left( \frac{F(\omega_v)}{p} - F(\omega_v) \right) + \tau \left( 1 - \frac{F(\omega_v)}{p} \right).$$

Hence $U(\emptyset, v) > U(\Omega^2, v)$ iff

$$\frac{(\tau - \zeta)(J(n,p)p - p(F(\omega_v) + 1) + F(\omega_v))}{p} > 0,$$

which holds if.

$$F(\omega_v) > \frac{p}{1-p}(1 - J(n,p)) = G(n,p). \tag{6}$$

We now state two claims, the proofs of which are given in the Appendix:

**Claim 2.** *For all $p$ below $\frac{1}{2} + \frac{1}{n+1}$, any critical point in $p$ of $G(\cdot, p)$ is a strict local maximum and any critical point in $p$ above $\frac{1}{2} + \frac{1}{n+1}$ is a strict local minimum.*

**Claim 3.** $\frac{dG(n,p)}{dp}|_{p=1/2} < 0$ *for all* $n \geq 7$.

Now, note that when $p = 1/2$, then $J(n, 1/2) = 1/2$ for all $n$ and hence $G(n, 1/2) = 1/2$. From Claim 2 and Claim 3 it follows that there does not exist any $p \in (1/2, (1/2)(1 + \frac{2}{n+1}))$ such that $G(n,p) \geq 1/2$. From Claim 2 it follows that when $n \geq 7$, any critical value of $G(n,p)$ for $p$ above $(1/2)(1 + \frac{2}{n+1})$ must correspond to a strict local minimum. Since $G(n,1) = 0$, it must therefore be that $G(n,p) < 1/2$ when $p \in [(1/2)(1 + \frac{2}{n+1}), 1)$. Therefore we have shown that when $n \geq 7$, $G(n,p) < 1/2$ for all $p \in (1/2, 1)$.

Since $F(\omega_v) \in (1/2, 1)$, it follows that when $n \geq 7$, the condition $F(\omega_v) > G(n,p)$ holds for all $p \in (1/2, 1)$ and therefore from inequality (6) we have $U(\emptyset, v) > U(\Omega^k, v)$ when the most influential persuasion strategy is considered. This proves part (b.i) of the proposition.

To prove part (b.ii), start with $n = 3$. Note that $G(3, 1/2) = 1/2$, $\frac{\mathbf{d}G(3,p)}{\mathbf{d}p}|_{p=1/2} = 1/2 > 0$, and that in the range $p \in (1/2, 1)$, the equation $\frac{\mathbf{d}G(3,p)}{\mathbf{d}p} = -6p^2 + 2p + 1 = 0$ yields a unique solution given by $p_3^* = \frac{\sqrt{7}}{6} + \frac{1}{6} > 1/2$. Also note that $\frac{\mathbf{d}^2 G(3,p)}{\mathbf{d}p^2} = -12p + 2 < 0$ for all $p \in (1/2, 1)$. Hence the maximum value of $G(3,p)$ is $G(3, p_3^*) = \frac{7\sqrt{7}}{54} + \frac{5}{27} = k^*(3)$ which is greater than $1/2$ and less than $1$. Therefore it follows from (6) that when $F(\omega_v) > k^*(3)$, then ex-ante voter welfare is always higher in the absence of an expert. This proves part (b.ii.1) of the proposition for $n = 3$.

Suppose $1/2 < F(\omega_v) < k^*(3)$. Since $\frac{\mathbf{d}^2 G(3,p)}{\mathbf{d}p^2} < 0$ for all $p \in [1/2, 1]$, it follows that there exists $F(\omega_v) < \hat{p}(3) < p_3^*$ such that for all $p \in (F(\omega_v), \hat{p}(3))$, the inequality $G(3,p) < F(\omega_v)$ holds while for all $p \in (\hat{p}(3), p_3^*)$, the inequality $G(3,p) > F(\omega_v)$ holds. Since $G(3, p_3^*) > 1/2$ and $G(3,1) = 0$ it follows that there exists $p_3^* < \tilde{p}(3)$ such that for all $p \in [p_3^*, \tilde{p}(3))$, the inequality $G(3,p) > F(\omega_v)$

holds while for all $p \in (\tilde{p}(3), 1)$, the inequality $G(3,p) < F(\omega_v)$ holds. This proves part (b.ii.2) of the proposition for $n = 3$.

Now consider $n = 5$. Note that $G(5, 1/2) = 1/2$, $\frac{\mathbf{d}G(5,p)}{\mathbf{d}p}|_{p=1/2} = \frac{1}{8} > 0$, and that in the range $p \in (1/2, 1)$, the equation $\frac{\mathbf{d}G(5,p)}{\mathbf{d}p} = 30p^4 - 36p^3 + 3p^2 + 2p + 1 = 0$ yields a unique solution given by $p_5^* = \frac{(548-30\sqrt{290})^{\frac{1}{3}}}{30} + \frac{(30\sqrt{290}+548)^{\frac{1}{3}}}{30} + \frac{1}{15}$. Note that $1/2 < p_5^* < p_3^*$ and $1/2 < G(5, p_5^*) < G(3, p_3^*) < 1$. Let $G(5, p_5^*) = k^*(5)$. When $F(\omega_v) > k^*(5)$, it follows from inequality (6) that ex-ante voter welfare is higher in the absence of an expert. This proves part (b.ii.1) of the proposition for $n = 5$. Suppose $1/2 < F(\omega_v) < k^*(5)$. Since $\frac{\mathbf{d}^2G(5,p)}{\mathbf{d}p^2}|_{p=1/2} < 0$, $\frac{\mathbf{d}^2G(5,p)}{\mathbf{d}p^2}|_{p=p_5^*} < 0$, and the equation $\frac{\mathbf{d}^2G(5,p)}{\mathbf{d}p^2} = 0$ is not solved for $p \in [1/2, p_5^*]$, it follows that $\frac{\mathbf{d}^2G(5,p)}{\mathbf{d}p^2} < 0$ for all $p \in [1/2, p_5^*]$. This proves the existence of $\hat{p}(5)$. Since $G(5, p_5^*) > 1/2$ and $G(5, 1) = 0$, the existence of $\tilde{p}(5)$ is proved. This proves part (b.ii.2) of the proposition for $n = 5$ and concludes the proof.

□

When the signal strength is low ($p < F(\omega_v)$) each member votes for $X$ for all states of the world irrespective of their private signals and irrespective of the number of voters (see Lemma 1). This rationalizes the expert's decision of not transmitting any information to the members. As a result, the ex-ante welfare is the same whether or not an expert is present.

But when the signal strength is high ($p > F(\omega_v)$), welfare is higher without persuasion, particularly when the committee size is sufficiently large ($n \geq 7$). The reason is as follows. In the presence of an expert, the nature of information provided is such that the members invariably vote for their less preferred alternative ($X$) when $\omega \in (\omega_v, \omega^*]$. However, they surely vote for their preferred alternatives $X$ (when $\omega \in [0, \omega_v]$) and $Y$ (when $\omega \in (\omega^*, 1]$). On the other hand without persuasion each voter votes according to his own signal for all $\omega \in [0, 1]$. In this case, the probability of a correct decision rises as the number of members increase (see discussion right after Lemma 1). Hence the relative advantage of the presence of an expert with respect to member welfare diminishes as the size of the committee rises. When $\omega \in (\omega_v, \omega^*]$, (which is the zone of incorrect decision-making due to persuasion), it follows by the same logic that the preferred alternative ($Y$) in this range would be the committee decision in the no expert case as the size of the committee goes up. Hence the welfare without persuasion is greater. What is novel in this result is that the critical committee size is $n = 7$.

Now consider the case when $n = 3$ or 5. In this case the probability of a correct decision without persuasion even when votes are according to private signals is low. It is here that the analysis gets interesting. To see this consider the following example. Suppose $F(\omega_v)$ is a uniform and let $n = 3$. It follows that in this case, if $k^*(3) \approx .527 < \omega_v < p$, then the ex-ante committee welfare is always higher without persuasion. Since the prior distribution is such that for the majority of the states the preferred alternative of the expert and the members coincide, the expert uses this as a leverage to manipulate the members by providing reliable information in such a way that the members would have been better-off without the prospect of receiving expert advice particularly since $p > F(\omega_v)$. Now suppose $1/2 < \omega_v = .51 < .527$. When $p \in (.51, \hat{p}(3) \approx .52197)$ or $p \in (\tilde{p}(3) \approx .68804, 1]$, the ex-ante committee welfare is higher without persuasion, while for $p \in (.52197, .68804)$ it is higher with persuasion. To understand this non-monotonic result note that when $F(\omega_v) < p$, the length of the zone of manipulation $(\omega_v, \omega^*]$ over which the expert is able to induce the members into voting for their less preferred choice decreases as the strength $p$ of their private signal rises. This means when $p$ is relatively low the zone of manipulation with expert advice is large, so that the welfare of the committee members is higher without persuasion. On the other hand, if $p$ is very high, the probability that the members will collectively choose the preferred alternative without persuasion is high for all states, and hence higher member welfare warrants absence of persuasion. However, for an intermediate range of $p$, the persuasion (yielding an advantage in the form of a guarantee that the committee decision will be the most preferred one for the members in all states barring when $\omega \in (\omega_v, \omega^*])$ becomes relatively more desirable, such that the ex-ante committee welfare is higher with persuasion.

## 4.2 Welfare with large bias: $F(\omega_v) < 1/2$

When the bias is large, our analysis in Section 3.2 indicates the crucial role of condition (*). It follows that the welfare consequence of persuasion will also depend upon that condition, and particularly so for the case (see Lemma 1) when $p > 1 - F(\omega_v)$. We have following proposition in this regard.

**Proposition 4.** *Public persuasion with large bias has the following welfare consequences:*

*(a) Let $1 - F(\omega_v) < p$. If $(*)$ is satisfied, then ex-ante member welfare is higher with persuasion iff $F(\omega_v) < \frac{p}{1-p}(1 - J(n,p))$. If $(*)$ is violated, then the ex-ante member welfare is always higher with persuasion.*

*(b) Let $p < 1 - F(\omega_v)$. In this case persuasion always corresponds to higher ex-ante welfare for the members.*

*Proof.* Let $1 - p < F(\omega_v) < \frac{1}{2}$. From Lemma 1 it follows that in this case the ex-ante welfare of the voter in the absence of the expert is given by

$$U(\emptyset, v) = J(n,p)\tau + (1 - J(n,p))\zeta \tag{7}$$

Suppose the expert is present, and the most influential persuasion strategy is considered. From Proposition 2.a.i it follows that when $(*)$ holds, then

$$U(\Omega^2, v) = \tau F(\omega_v) + \zeta \left( \frac{F(\omega_v)}{p} - F(\omega_v) \right) + \tau \left( 1 - \frac{F(\omega_v)}{p} \right) \tag{8}$$

It follows that in this case $U(\emptyset, v) > U(\Omega^k, v)$ if (6) holds.
Suppose $(*)$ is violated, in which case from Proposition 2.a.ii it follows that

$$U(\Omega^2, v) \;\; = \;\; \tau \left( \frac{F(\omega_v)}{p} - \frac{1-p}{p} \right) + \left( 1 - \left( \frac{F(\omega_v)}{p} - \frac{1-p}{p} \right) \right) (J(n,p)\tau + (1 - J(n,p))\zeta) \; .$$

Since $p > 1 - F(\omega_v)$, it follows that $0 < \frac{F(\omega_v)}{p} - \frac{1-p}{p} < 1$. This along with $\tau > \zeta$, $0 < J(n,p) < 1$ implies that for this case $U(\Omega^2, v) > U(\emptyset, v)$. This proves Part (a) of the proposition.
Now consider $p < 1 - F(\omega_v)$. It follows from Lemma 1 that

$$U(\emptyset, v) = F(\omega_v)\zeta + (1 - F(\omega_v))\tau$$

From Proposition 2 part (b.i) it follows that in the presence of public signal generated from most influential persuasion strategy, when $n \geq 5$, the ex-ante welfare of the voter is given by expression (8). Hence it follows that $U(\Omega^k, v) > U(\emptyset, v)$ if

$$(\tau - \zeta) \left( 2F(\omega_v) - \frac{F(\omega_v)}{p} ) \right) > 0 \tag{9}$$

which always holds for all $1/2 < p < 1$. Hence in this case the presence of the expert leads to higher voter welfare. When $n = 3$, and $p \in (1/2, p')$ where $p'$ is defined in Proposition 2 part (b.ii), it is analogously shown that (9) holds and presence of the expert leads to higher voter welfare. When $n = 3$, and $p \in (p', 1)$, it follows that under presence of the provision of expert advice under most influential persuasion strategy, the ex-ante voter welfare is given by

$$U(\Omega^2, v) \;\; = \;\; \left( \frac{F(\omega_v)}{1-p} \right) (J(n,p)\tau + (1 - J(n,p))\zeta) + \left( 1 - \frac{F(\omega_v)}{1-p} \right) \tau.$$

Hence it follows that $U(\Omega^2, v) > U(\emptyset, v)$ if

$$\frac{F(\omega_v)(\tau - \zeta)(J(n,p) - p)}{1-p} > 0$$

which always holds since $F(\omega_v) > 0$, $\tau > \zeta$ and $p < J(n,p)$. Hence in this case the presence of an expert leads to higher ex-ante voter welfare. This proves part (b) of the proposition and concludes the proof.

□

Consider part (a) of Proposition 2, where $1 - F(\omega_v) < p$ and each member votes according to his private signal in the absence of expert advice. Consider the first scenario where condition (*) is satisfied. As discussed earlier, $(*)$ is always satisfied when the number of committee members goes to infinity. In this case, ex-ante committee welfare is higher in the absence of expert advice. This is because of the following: in the expert's presence the probability of a correct decision is invariant to the number of committee members since each of them follows a signal invariant (or pooling) voting strategy where the decision is always unanimous. In the expert's absence the members vote according to their private signals which implies that the large number of private signals aggregated to form the social decision guarantees that the correct social decision will be arrived at with a very high probability. Thus for a very large committee, the presence of a manipulative expert hurts welfare.

If however, the size of the committee is small, then whether the presence of an expert leads to higher ex-ante welfare or not depends on the 'relative size' of the set of states for which decision-making is improved owing to the information provided by the expert vis a vis the set of states over which the members are manipulated to vote for their less favored alternative. This in turn is compared to the scenario where the expert is absent. Both the cases where expert presence or absence is desirable for higher ex-ante welfare is feasible, which we demonstrate by citing two examples where the number of members is low and $(*)$ holds.

**Example 1** (Condition $(*)$ holds, expert presence improves welfare.)**.** For the first example, let $F(\omega_v) = .35$, $p = .66$ and $n = 3$. It is already shown earlier that $(*)$ is satisfied for these values of the parameters. For these values, $\frac{p}{1-p}(1 - J(n, p)) \approx .521 > .35 = F(\omega_v)$, and hence in this case the presence of the expert leads to higher ex-ante welfare of the committee members.

**Example 2** (Condition $(*)$ holds, expert presence hurts welfare.)**.** Let $F(\omega_v) = .48$, $p = .74$ and $n = 3$. In this case the conditions $\frac{1}{3} < F(\omega_v) < \frac{1}{2}$ and $1 - F(\omega_v) < p \leq \frac{1+F(\omega_v)}{2}$ holds, and hence $(*)$ is valid which is checked by noting that corresponding to these values, $(Q(n, 1; \omega_v) - 1) \approx -.083 < 0$. However, in this case $\frac{p}{1-p}(1 - J(n, p)) \approx .477 < .48 = F(\omega_v)$, and hence in this case the absence of the expert leads to higher ex-ante voter welfare.

Now consider the case where $(*)$ is violated, an example of which has been provided earlier. In this case, due to transmission of information by the expert, the members follow a signal invariant strategy of voting $X$ for a certain range of states contained entirely in $[0, \omega_v]$, which is their preferred alternative for these states of the world. For the rest of the states the members vote according to their private signals, which they would have done anyway even in the absence of the expert. Hence ex-ante committee welfare is higher in the presence of the expert when $(*)$ is violated.

Now consider part (b) of the proposition, where the precision of signals of the members is sufficiently low such that in the expert's absence the members vote for $Y$ irrespective of their private signal received. Given the prior distribution, the states where the preferred alternative of the expert and the committee members are different are more likely to occur. Therefore the power of the expert to manipulate the members is limited, and thus the welfare of the members is higher if expert advice is received.

# 5 Conclusion

In this paper we study the effect of transmission of public information through a persuasion strategy chosen by a biased expert to partially informed committee members. We show that public persuasion never hurts welfare if the precision of private signals received by the committee members is low. Otherwise, public persuasion will hurt welfare for large committees. The strategically chosen content of the equilibrium persuasion strategy overpowers the private information of the members and invariably makes them vote for a particular alternative. In contrast, without persuasion the members would have voted according to their private signals so that the probability of the correct decision increases with the size of the committee. Hence absence of expert advice can improve welfare in large constituencies. This perverse effect of presence of additional information in the form of expert advice can also appear in small constituencies, though not universally.

It is important to note that while the possibility of reduction in welfare owing to additional public information in our model is driven by the presence of strategic i.d.t chosen by the expert, similar public information can appear from non-strategic agents as well, or even from sources whose intention is to send as much information as possible under the belief that more information cannot hurt the probability of correct decision. This is specially true in the case of court trials where the judiciary tries to ensure that the trial is as informative as possible. Yet, since not all trials can reveal the truth with certainty, one may ask if and when any additional (but incomplete) public information adversely affects the probability of making correct judgments. Our results indicate that when the trial is known to provide partially informative slants with evidential input, the probability that a jury takes the correct decision can go down, though such trials are always welcome if the jury is replaced by a dictatorial judge. Among other extensions, one may also investigate how the results are altered if we consider a multiple alternatives voting model and/or apply other aggregation rules such as the system of approval voting or cumulative voting. It is also quite natural to introduce multiple experts with either like or conflicting biases, and examine their implication on committee welfare. We reserve these for future research.

# 6 Appendix

*Proof of Lemma 1* : Consider the case where a voting strategy $v_j = s_j$ is followed by all $j \in I, j \neq i$. Let $E(n-1,k)$ be the event that out of $n-1$ private signals, exactly $k$ equal $X$, $k = 0, \cdots, n-1$. Voter $i$ is pivotal if and only if $k = \frac{n-1}{2}$. We shall use the shorthand $Piv_i = E\left(n-1, \frac{n-1}{2}\right)$. Let

$$\tilde{\gamma}_{s_i} = \mathbb{P}[\omega \leq \omega_v | Piv_i, s_i] = \frac{A}{B}$$

where

$$A = \mathbb{P}[Piv_i|\omega \leq \omega_v]\mathbb{P}[s_i|\omega \leq \omega_v]\mathbf{Pr}[\omega \leq \omega_v]$$

and

$$B = \mathbb{P}[Piv_i|\omega \leq \omega_v]\mathbb{P}[s_i|\omega \leq \omega_v]\mathbf{Pr}[\omega \leq \omega_v] + \mathbb{P}[Piv_i|\omega_v < \omega]\mathbb{P}[s_i|\omega_v < \omega]\mathbb{P}[\omega_v < \omega]$$

Note that

$$\mathbb{P}[Piv_i|\omega \leq \omega_v] = \mathbb{P}[Piv_i|\omega > \omega_v] = \binom{n-1}{\frac{n-1}{2}} p^{\frac{n-1}{2}}(1-p)^{\frac{n-1}{2}}$$

Given the prior density $f(\omega)$ with the associated distribution $F(\omega)$,

$$\tilde{\gamma}_X = \frac{pF(\omega_v)}{pF(\omega_v) + (1-p)(1-F(\omega_v))}$$

and

$$\tilde{\gamma}_Y = \frac{(1-p)F(\omega_v)}{(1-p)F(\omega_v) + p(1-F(\omega_v))}.$$

Note that under the simple majoritarian aggregation rule, $\tilde{\gamma}_{s_i} = \gamma_{s_i}$, where $\gamma_{s_i} = \mathbb{P}[\omega \leq \omega_v | s_i]$. Fix any symmetric voting strategy profile $v_{-i}$. In the absence of the expert, let the expected utility of the voter $i$ from voting $v_i$ given that he receives a private signal of $s_i$ and the rest of the members are following a voting strategy $v_{-i}$ be denoted by $U_{v_i}(\emptyset, v_{-i}, s_i)$. Define

$$U_X(\emptyset, v_{-i}, s_i) = \sum_{k=0}^{n-1} \left[ \mathbb{P}[E(n-1,k)|s_i] \left( \int_{\omega \in \Omega} u(\delta(v_{-i}, v_i = X), w) f(\omega|s) \right) d\omega \right],$$

and

$$U_Y(\emptyset, v_{-i}, s_i) = \sum_{k=0}^{n-1} \left[ \mathbb{P}[E(n-1,k)|s_i] \left( \int_{\omega \in \Omega} u(v_{-i}, v_i = Y), w)) f(\omega|s) \right) d\omega \right].$$

At this voting strategy profile $v_{-i}$, voter $i$ votes for $X$ if and only if $U_X(\emptyset, v_{-i}, s_i) > U_Y(\emptyset, v_{-i}, s_i)$. This reduces to $(2\tilde{\gamma}_{s_i} - 1)\tau > (2\tilde{\gamma}_{s_i} - 1)\zeta$. Since $\tau > \zeta$, the above inequality holds if and only if $(2\tilde{\gamma}_{s_i} - 1) > 0$, which implies $\tilde{\gamma}_{s_i} > 1/2$.

Hence for a strategy profile $v$ where $v_i(\emptyset, X) = X$ and $v_i(\emptyset, Y) = Y$ for any $i \in I$ to hold in equilibrium, both the conditions $\tilde{\gamma}_X > 1/2$ and $\tilde{\gamma}_Y < 1/2$ need to be satisfied.

Now consider the scenario where either of the condition $\tilde{\gamma}_X > 1/2$ and $\tilde{\gamma}_Y < 1/2$ is violated, in which case it follows that a voting strategy $v_i = s_i$ cannot be sustained in a symmetric equilibrium. In this case we consider other possible symmetric equilibria which are (i) $v_i = X$ for each $s_i \in S$ for all $i \in I$ or (ii) $v_i = Y$ for each $s_i \in S$ for all $i \in I$. Note that if for all $j \in I, j \neq i$, the voting strategy $v_j = X$ for each $s_j \in S$ is followed, then voter $i$ is never pivotal. Furthermore, for this case $\mathbb{P}(w \leq w_v | v_{-i}, s_i) = \mathbb{P}(w \leq w_v | s_i)$. In an informative equilibrium, since the preference of the voter is $X \succ Y$ if $\omega \leq \omega_v$ and $Y \succ X$ if $\omega > \omega_v$, upon receiving a private signal $s_i = X$ the voter $i$ votes $v_i = X$ if $\gamma_X > 1/2$ and $v_i = Y$ if $\gamma_X < 1/2$. Similarly, upon receiving a private signal $s_i = Y$ the voter $i$ votes $v_i = X$ if $\gamma_Y > 1/2$ and $v_i = Y$ if $\gamma_Y < 1/2$.

Consider $F(\omega_v) > 1/2$. In this case the condition $\gamma_X > 1/2$ implies $p > 1 - F(\omega_v)$, which always holds since $p > 1/2$. The condition $\gamma_Y < 1/2$ implies $p > F(\omega_v)$. The binding condition for $v_i(\emptyset, s_i) = X$ for each $s_i \in S$ is therefore $p > F(\omega_v)$. Note that when $p < F(\omega_v)$, then $\gamma_X > 1/2$ and $\gamma_Y > 1/2$. This proves part (a) of the lemma.

Now consider $0 < F(\omega_v) < 1/2$. In this case the condition $\gamma_Y < 1/2$ implies $p > F(\omega_v)$, which always holds since $F(\omega_v) < 1/2 < p$. The condition $\gamma_X > 1/2$ implies $p > 1 - F(\omega_v)$, which is therefore the binding condition for $v_i(\emptyset, s_i) = s_i$ for each $s_i \in S$. Note that when $p < 1 - F(\omega_v)$, then $\gamma_Y < 1/2$ and $\gamma_X < 1/2$ holds. This proves part (b) of the lemma and concludes the proof.

*Proof of Claim 1* :
We can express

$$J(n+2,p)-J(n,p) \quad = \quad \binom{n}{\frac{n-1}{2}} p^{\frac{n+5}{2}}(1-p)^{\frac{n-1}{2}}\left(\frac{1-p}{p}-\left(\frac{1-p}{p}\right)^2\right)$$

Hence the sufficient condition for $J(n+2,p)-J(n,p)>0$ is $\frac{1-p}{p}-(\frac{1-p}{p})^2>0$, which holds in our model since $p\in(1/2,1)$. This proves the claim.

*Proof of Claim 2* : Let $L_{n,j}(p)\equiv\binom{n}{j}p^j(1-p)^{n-j}$. Also let

$$F_{n,r}(p)\equiv\sum_{j=0}^{r}\binom{n}{j}p^j(1-p)^{n-j}=\sum_{j=0}^{r}L_{n,j}(p).$$

Since $n$ is an odd integer, and let $m$ be an even integer given by $m=\frac{n+1}{2}$. We may express

$$G(n,p)\equiv\frac{p}{(1-p)}F_{n,m-1}(p)$$

At $p=0$ we replace this definition by its limiting value $G(n,0)=0$. We also have $G(n,1)=0$. Note that

$$\frac{d}{dp}L_{n,j}(p)=n\left(L_{n-1,j-1}-L_{n-1,j}\right) \tag{10}$$

So

$$\frac{d}{dp}F_{n,m-1}(p)=\sum_{j=0}^{m-1}n\left(L_{n-1,j-1}-L_{n-1,j}\right)=-nL_{n-1,m-1}.$$

Hence we have

$$\frac{d}{dp}G(n,p)=\frac{1}{(1-p)^2}F_{n,m-1}(p)-\frac{np}{(1-p)}L_{n-1,m-1} \tag{11}$$

Differentiating both sides of (11) again with respect to $p$ we have

$$\frac{d^2}{dp^2}G(n,p)=\frac{2}{(1-p)^3}F_{n,m-1}(p)-\frac{2n}{(1-p)^2}L_{n-1,m-1}-n\left(\frac{1}{(1-p)}-1\right)L'_{n-1,m-1}$$

From (10) we have

$$L'_{n-1,m-1} = (n-1)\left(L_{n-2,m-2} - L_{n-2,m-1}\right),$$

and hence

$$\frac{d^2}{dp^2}G(n,p) = \frac{2}{(1-p)^3}F_{n,m-1}(p) - \frac{2n}{(1-p)^2}L_{n-1,m-1} - n(n-1)\frac{p}{(1-p)}\left(L_{n-2,m-2} - L_{n-2,m-1}\right) \tag{12}$$

By the Weierstrass theorem, $G(n,p)$ has at least one global maximum over $[0,1]$.

We now require $G'(n,p^*) = 0$, so from (11) we have

$$\frac{1}{(1-p^*)^2}F^*_{n,m-1} - \frac{np^*}{(1-p^*)}L^*_{n-1,m-1} = 0$$

where $F^*_{n,m-1} \equiv F_{n,m-1}(p^*)$ etc.

For maximization we require $G''(n,p^*) \leq 0$, so from (12) we have

$$\frac{2}{(1-p^*)^3}F^*_{n,m-1} - \frac{2n}{(1-p^*)^2}L^*_{n-1,m-1} - n(n-1)\frac{p^*}{(1-p^*)}\left(L^*_{n-2,m-2} - L^*_{n-2,m-1}\right) \leq 0$$

Using the first-order condition, the second order condition is equivalent to

$$-2n(1-p^*)L^*_{n-1,m-1} - n(n-1)p^*(1-p^*)\left(L^*_{n-2,m-2} - L^*_{n-2,m-1}\right) \leq 0,$$

which is further simplified as, so the SOC is equivalent to

$$2 + (n-1)p^*\left(\frac{m-1}{n-1}\frac{1}{p^*} - \frac{n-m}{n-1}\frac{1}{(1-p^*)}\right) \geq 0$$

$$\Leftrightarrow m+1 \geq (n+1)p^*.$$

Using $n = 2m-1$ we can write the above inequality as $p^* \leq \frac{m+1}{2m}$, which implies that

$$p^* \leq (1/2)\left(1 + \frac{1}{m}\right). \tag{13}$$

Thus it follows from (13) that any critical point below $(1/2)\left(1+\frac{1}{m}\right)$ is a strict local maximum and any critical point above is a strict local minimum. Putting $m = \frac{n+1}{2}$ proves the claim.

*Proof of Claim 3* : Note that

$$\frac{\mathbf{d}G(n,p)}{\mathbf{d}p} = \frac{1}{(1-p)^2} - \sum_{j=\frac{n+1}{2}}^{n} \binom{n}{j} (p)^j (1-p)^{n-j} \left[ \frac{j+1}{1-p} - \left( \frac{p}{(1-p)^2} \right) (n-j-1) \right]$$

Hence

$$\frac{\mathbf{d}G(n,p)}{\mathbf{d}p}|_{p=1/2} = 4 \left[ 1 - \left( \frac{1}{2} \right)^{n+1} \sum_{j=\frac{n+1}{2}}^{n} \binom{n}{j} (2j+2-n) \right]$$

We want to establish that for $n \geq 7$, $\frac{\partial G(n,p)}{\partial p}|_{p=1/2} < 0$ which implies

$$2^{n+1} < \sum_{j=\frac{n+1}{2}}^{n} \binom{n}{j} (2j+2-n) \tag{14}$$

Note that $\sum_{j=0}^{n} \binom{n}{j} = 2^n$, and since $n$ is odd, we also have

$$\sum_{j=\frac{n+1}{2}}^{n} \binom{n}{j} = (1/2) \sum_{j=0}^{n} \binom{n}{j}.$$

Using these, we may express (14) as

$$2^n < \sum_{j=\frac{n+1}{2}}^{n} \binom{n}{j} (2j-n) \tag{15}$$

Let $k = \frac{n-1}{2}$. We may express (15) as

$$2 \left[ \binom{2k+1}{k+1} + \binom{2k+1}{k+2} + .. + 1 \right] < \binom{2k+1}{k+1} + 3\binom{2k+1}{k+2} + 5\binom{2k+1}{k+3} + .. + (2k+1)$$

which holds if

$$2 \left[ \binom{2k+1}{k+1} + \binom{2k+1}{k+2} + \binom{2k+1}{k+3} \right] < \binom{2k+1}{k+1} + 3\binom{2k+1}{k+2} + 5\binom{2k+1}{k+3}$$

Using the relation

$$\binom{n}{j+1} = \binom{n}{j} \frac{n-j}{j+1},$$

the above inequality reduces to

$$\frac{k}{k+2}+\frac{3k(k-1)}{(k+2)(k+3)}>1$$

Consider

$$g(k)=\frac{k}{k+2}+\frac{3k(k-1)}{(k+2)(k+3)}-1.$$

Note that for $k>0$, the equation $g(k)=0$ has a unique solution at $k=\frac{\sqrt{97}}{6}+\frac{5}{6}<3$, and $\frac{\mathbf{d}g(k)}{\mathbf{d}k}|_{k=\frac{\sqrt{97}}{6}+\frac{5}{6}}>0$. Hence $g(k)>0$ for all $k\geq 3$, which implies (15) is satisfied for all $n\geq 7$. This proves Claim 3.

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