Committees with leaks

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Abstract

We analyse the quality (informativeness and efficiency) of advice obtained from a committee of careerist experts where voting is secret but voting profiles are 'leaked' with an exogenously given probability. We show that fully informative voting is generically possible only when the common prior is not too informative, the committee uses the unanimity rule and faces the possibility of leakage. It is then shown that informativeness and efficiency are mutually exclusive properties of expert committees.
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1 Introduction

The relative quality of expert advice across fully transparent and fully secretive committees has been the subject of a number of papers.[1] Our objective is to look into the performance of secretive committees that face an exogenously given probability with which the recommendations of its members are collectively 'leaked' to an evaluator (EV) of expertise. Such committees will be called *committees under leakage threats*.[2] When experts care only

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[1]See for example Gersbach and Hahn (2001) and (2011), Sibert (2003), Fingleton and Raith (2005), Levy (2007a), Meade and Stasavage (2008), Swank and Visser (2010) and Seidmann (2010).

[2]An example of the scenario we motivate could be the recent case involving the Federal Open Market committee (FOMC) transcripts that were initially considered to be confidential but ended up being public. The FOMC meetings were secretly videotaped up to a point in time, and transcripts based on these recordings were released later on April 9th and 10th, 2013. Other examples include the various instances of leakage conducted by organisations such as Wikileaks, that have caused a lot of controversy in recent times.

for the EV's beliefs regarding their individual expertise (i.e., they are pure careerists) we show (see Theorem 1) that to obtain informative voting (i.e. voting in accordance to one's updated beliefs) as an equilibrium outcome it is necessary and sufficient to have a committee that (a) uses the unanimity voting rule and faces a leakage threat and (b) the common prior is not too informative. We then show (see Theorem 2) that informative voting hurts social welfare.

Gersbach and Hahn (2001) were the first to explore the dichotomy between information acquisition and optimality in expert committees. They compared a fully transparent and a fully secretive committee in a dynamic model and found that a secretive process allows for better decisions while a transparent process leads to better identification of the talents of the experts. Sibert (2003) similarly argues that secretive processes yield better decisions by reducing the incentive of agents to distort their actions to signal their types. In our framework, the tendency to distort actions is present both in a fully secretive as well as a fully transparent committee. Our model is closely related to the work by Levy (2007a). She compares a fully transparent with a fully secretive committee to show that careerist experts exhibit conformist (herding) or non-conformist (anti-herding) tendencies so that their votes may not be informative or efficient in either environment.[3] Given this, our result that leakage threats yield non-distorted advice is new. Levy (2007a) also finds that if the common prior is sufficiently informative and biased towards the status quo alternative, a secretive committee with the unanimity rule induces the highest level of welfare. We show that this is not a local result as it is robust to the possibility that transparency can be random (and of any degree). Moreover, Levy (2007a) does not report welfare results when the prior is not too biased (that is largely uninformative). We show that for such cases, the opposite result holds where secrecy hurts welfare and it is optimal to have perfectly transparent committees if one is confined to the unanimity rule. Interestingly, we also find that in every such situation of low prior, the simple majority rule yields the maximum welfare.[4]

[3]The possibilities of herding and anti-herding have been explored in earlier works by Holmstrom (1999), Scharfstein and Stein (1990), Zwiebel (1995) and Ottaviani and Sorensen (2006a and 2006b), although these works do not address voting in committees.

[4]In Levy (2007b), the analysis was confined to two member committees, and therefore the welfare implications of the simple majority rule could not be addressed.

2 The Model

There are two possible actions A and B. Information about which should be the 'correct choice' is available from three equally salient and independent sources (or *dimensions*), called $1, 2$ and 3. The true *state* in dimension i is $w_i \in W = \{a, b\}$, $i = 1, 2, 3$, with the following interpretation: B is the *correct action according to dimension i* if and only if $w_i = b$. Let $\mathcal{W} = \{a, b\}^3$ with $w = (w_1, w_2, w_3) \in \mathcal{W}$ being a state vector. Let $\pi = \Pr[w_i = b] > 1/2$ be the *common prior* for each dimension i.[5] The choice of an action is swayed by the decision of a *committee* composed of three *experts* called $i = 1, 2, 3$. Expert i is proficient exclusively in dimension i and receives a private signal $s_i \in S = \{a, b\}$ about the true state w_i in his dimension of expertise. The informative precision, denoted by t_i, of the signal s_i is called expert i's *talent*, with $t_i \in T = [1/2, 1]$ so that $\Pr[s_i = a | w_i = a] = \Pr[s_i = b | w_i = b] = t_i$. Expert i's talent is his private information and it is common knowledge that t_i is *uniformly and independently* distributed over the support T.

Expert i provides an *advice* $m_i \in M = \{a, b\}$ simultaneously and independently along with the other experts and the advice (or simply *vote*) a (likewise b) is construed as the pronouncement that "a (likewise b) is the true state according to dimension i". Denote by $\mathcal{M} = \{a, b\}^3$ the set of vote profiles with $m = (m_1, m_2, m_3) \in \mathcal{M}$. The *decision* of the committee is denoted by d_x and is defined by the *voting rule* $x \in X = \{2, 3\}$ as follows: $d_x : \mathcal{M} \to \{A, B\}$ such that $d_x = A$ if and only if $|\{i : m_i = a\}| \geq x$. If $x = 2$ we call this *majority* while if $x = 3$ we call this *A-unanimity* (or, unanimity in short). The voting rule x is common knowledge.

There is an evaluator (EV) whose goal is to estimate the individual talents of the three experts. The true states w_i, $i = 1, 2, 3$, are revealed to the EV after all votes are cast and the committee decision d_x (that is always observed by the EV) is reached. A *committee* under the prospect of a possible leakage, denoted by the pair $C = (x, p)$, consists of a *secret committee* with voting rule x and an *exogenous probability* $p \in P = [0, 1]$ with which the vote profile $m = (m_1, m_2, m_3) \in \mathcal{M}$ is revealed to the EV. Expert i's (voting) *strategy* is a function $\sigma_i : T \times S \times P \times X \to M$ and let $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ be a strategy profile. Let $\zeta_i : T \times S \times P \times X \to M$ be the *conjecture* held by the EV about expert i's voting strategy with $\zeta = (\zeta_1, \zeta_2, \zeta_3)$. Let $\tau(m_i, w_i, \zeta) = \mathbb{E}(t_i | (m_i, w_i), \zeta)$ be the talent evaluation function which is simply the expectation held by the EV about the true value of expert i's talent, given his advice m_i, the observed state w_i and the conjecture ζ. The pay-off function of expert i is simply τ. We assume that each expert is an expected utility maximiser and

[5] Since $\pi > 1/2$, one may think of B as the "conventional" choice.

votes in order to maximise the *expected evaluation of his talent* τ held by the EV, where the expert's expectations are based solely upon his own signal, his talent and the common prior π.

The above environment leads to a Bayesian game of reputational cheap talk and as in Levy (2007a) we consider symmetric equilibria in pure strategies where the following hold: (i) each expert votes as if they are pivotal at each event where their individual votes are not revealed to the EV,[6] (ii) the EV's conjecture coincides with the voting strategies followed by the experts and (iii) updating of beliefs follows the Bayes' rule wherever possible. Let $v(\pi, t_i|s_i)$ be the *posterior* probability-belief held by an expert with talent t_i who receives a private signal s_i that $w_i = a$. We say that expert i's strategy σ_i is *informative* if σ_i implies the following: $m_i = b$ if and only if $v(\pi, t_i|s_i) \leq \frac{1}{2}$ (and $m_i = a$ otherwise). We say that a committee $C = (x, p)$ with leakage probability p and voting rule x is *informative* if there exists an equilibrium σ^* such that σ_i^* is informative for all $i = 1, 2, 3$. Finally, we define social welfare in a standard way to be the aggregate probability with which the committee decision is correct in as many dimensions as possible. Thus the social welfare under a committee $C = (x, p)$ with prior π is

$$W(x,p,\pi) = \sum_{w \in \mathcal{W}} \Pr(w) \left[\left(\Pr(d_x = A|w,x,\sigma) \sum_{k \in 1,2,3} I_k^a \right) + \left(\Pr(d_x = B|w,x,\sigma) \sum_{k \in 1,2,3} I_k^b \right) \right]$$

where, for $z \in \{a, b\}$ in dimension $k \in \{1, 2, 3\}$, we have $I_k^z = 1$ if $w_k = z$ and 0 otherwise. Welfare maximisation entails identifying an equilibrium that corresponds to the highest level of social welfare.

3 Quality of Advice

We begin by characterising informative committees. The following theorem shows that random transparency is a necessary feature of such committees for any generic value of the prior π.

Theorem 1. *Consider a committee $C = (x, p)$ with voting rule x and leakage probability p and let π be an arbitrary common prior. Then there exists a unique symmetric equilibrium with the following properties: (i) If $x = 2$ then for all $p \in [0, 1]$ and for all $\pi \in (1/2, 1)$, $C = (x, p)$ is not informative; (ii) If $x = 3$, then there exists $\tilde{\pi} \in (1/2, 1)$ such that (a) for each $\pi < \tilde{\pi}$, there*

[6] Pivotality is redundant for the events where the voting profile m is observed by the EV.

exists $p \in (0,1)$ such that $C(x,p)$ is informative, (b) for all $\pi > \tilde{\pi}$ and all $p \in [0,1]$, $C = (x,p)$ is not informative.[7]

Note that when $s_i = b$, both sources of information available to expert i (viz. π and s_i) suggest that $w_i = b$. Therefore, experts of all talent levels vote $m_i = b$ since they induce a higher τ by making a correct prediction rather than an incorrect one. With $s_i = a$, while one source (π) indicates that $w_i = b$ the other source (s_i) indicates that $w_i = a$. In this case there is non-conformism exhibited by mediocre experts who have an innate tendency to provide advice in favour of the unconventional choice A in the hope of being hailed by the EV to have more talent if they can correctly predict an outcome that was initially considered to be less possible (since $\pi > 1/2$). This is the only force at work when the committee is fully transparent ($p = 1$). Hence reducing the probability of transparency helps dampen non-conformist behaviour. When secrecy of the committees can be guaranteed ($p = 0$), one needs to look separately at the unanimous and the majority voting rules. Under the unanimity rule ($x = 3$) if i votes as-if pivotal and $m_i = a$, the committee decision will be A, and the EV will know that $m_i = a$. In this case, the non-conformist force will be at play again as it then becomes essentially a transparent committee. However, if $m_i = b$, then while the decision will be B, the EV will not be sure about i's vote. Interestingly therefore, experts can control opacity through their votes and hence mediocre experts tend to be conformists and vote for B to keep the committee secret. With these opposing forces at work, the proof of the theorem relies on purely quantitative aspects and we show that there are degrees of transparency under which the opposing forces of conforism and non-conformism counter-balance each other and guarantee informative voting. Now consider $x = 2$ under full secrecy ($p = 0$). Here, irrespective of whether the expert votes for a or b, he can never reveal his personal vote to the EV. Hence the conformist tendency present in a secretive unanimous committee is absent in a majoritarian committee, although the non-conformist tendency remains. So informative votes cannot be obtained with the majoritarian voting rule.

Our second result is on aggregate welfare where among other findings we show that informative committees are never welfare maximising.

Theorem 2. *Consider a committee $C = (x,p)$ with voting rule x and leakage probability p. Then, the following is true in equilibrium: (i) $C = (x,p)$ is informative if and only if it is not welfare maximising; (ii) When $x = 3$,*

[7] We also prove that for the special case where $\pi = \tilde{\pi}$, the committee $C = (3,0)$ yields informative voting.

there exists two threshold levels of the common prior, $1/2 < \pi^* \leq \pi^{**} < 1$ such that (a) for all $\pi < \pi^*$, aggregate welfare is maximised if and only if $p = 1$, and (b) for all $\pi > \pi^{**}$, aggregate welfare is maximised if and only if $p = 0$; and (iii) There exists a threshold level of the common prior $\hat{\pi} > 1/2$ such that for all $\pi < \hat{\pi}$, a fully transparent ($p = 1$) majoritarian ($x = 2$) committee is socially better than a unanimous ($x = 3$) committee with any transparency probability $p \in [0, 1]$.

To understand Part (i) and part (ii a), suppose $x = 3$ and the prior π is close to 1/2. Note that although the ex-post talent levels of the experts are heterogeneous with probability 1, the weights assigned to their votes are equal. Suppose two of the experts are highly talented while the third is mediocre. If the two highly talented experts vote for A and the mediocre expert votes for B, (all three voting in accordance to their true posterior probabilities), the decision of the committee is B. This can potentially hurt welfare, because even though the mediocre expert has voted informatively, the quality of his information itself is weak owing to his mediocre talent, and under $x = 3$ it proves decisive. Hence for a low prior, we require the relatively mediocre experts to exhibit non-conformist tendencies through insincere assertions and not vote informatively. This is achieved by having a fully transparent committee ($p = 1$) since an increase in p pushes the cutoff talent t^* towards 1/2. On the other hand, when π is high (as in part (ii b)), welfare maximisation warrants allowing only the very smart experts to cast votes against the prior when they receive a contrarian signal a, thereby minimising the chance of overturning the committee decision in favour of the unconventional state. This under unanimity is achieved in a fully secret committee ($p = 0$). That the dichotomy between informative voting and welfare maximisation holds for *all* levels of the prior is not obvious, making part (i) of Theorem 2 particularly noteworthy. We collect this observation below:

Corollary 1. *Informative Voting and Aggregate Welfare Maximisation through committee decisions are mutually exclusive objectives if experts care only about their individual reputations and have private information in independent spheres of expertise.*

Part (iii) of Theorem 2 shows that when the prior is low, a fully transparent committee using the simple majority rule is better in terms of aggregate welfare than any unanimity committee. This is due to the fact that the simple majority rule does not suffer from any bias in favour of either choices, whereas the unanimity rule is inherently biased in favour of the conventional choice B. Hence even a fully transparent ($p = 1$) majoritarian committee corresponds to a higher level of welfare than a secretive committee operating under any leakage threat p.

Remark 1 (Choice of transparency). *As mentioned earlier, the probability p with which vote profiles are 'leaked' to the EV is an exogenous element of the environment. Although there is strong evidence in recent times of committee designers trying to affect leakage probabilities through enhanced surveillance and strict punishment threats, we made the exogeneity assumption to highlight the fact that one may still have insufficient control over leakage. This can then be viewed as an important constraint on committee design that has been overlooked so far in the literature. And when faced with a known leakage threat p one may ask if full transparency (which can always be achieved by the designer through immediate publication of voting records) can be desirable in order to maximise welfare.*[8] *While a complete analysis of this novel mechanism design problem is computationally hard and beyond the scope of this short note, our analysis does provide partial answers for the case when the EV's prior information is largely uninformative so that the role of expert advice is most valuable. We show (in part (ii) of Theorem 2) that in this case irrespective of the leakage threat p, the designer will strictly prefer to make unanimous committees fully transparent rather than keeping them secretive.*

What if messages are direct reports of the private signals and the committee decision rule is applied on these reports? Since talents are private information, aggregate information content from truthful reporting (of private signals) in such a committee design cannot be more than what is achieved under informative voting in the model proposed by Levy (2007a) that we have used. This is because in our model each expert personally uses all his available sources of information (viz. private signal, talent level (which is never known by the EV) and the prior) to calculate the posterior probability and votes accordingly. Another possibility is direct reporting of posteriors. This would require an axiomatic approach since there is no obvious way to aggregate probability judgements and it is not clear how to define majority. An interesting question is whether there are reasonable axiomatic properties of probability aggregation rules where informative voting obtainable in equilibrium is also welfare maximising.

Remark 2 (B - Unanimity). *Although we have not explicitly analysed the voting rule $x = 1$ where the non-conventional choice A is made whenever at least one expert votes for it, it is easy to see that both in a transparent and a secretive committee there is a reputation driven bias towards voting for A. Hence an informative equilibrium will not be obtainable. It is also straightforward to see that when the prior π is close to 1, the A - unanimity voting rule ($x = 3$) will welfare dominate.*

[8] We thank an anonymous referee for suggesting this mechanism design aspect of the problem.

4 Questions and Extensions

By considering three experts in this binary set-up with B as the 'status quo' choice, we are able to compare between a procedurally biased (unanimity that is biased towards B) and an unbiased (simple-majority) voting rule. We have shown that the existence of procedural bias is crucial in obtaining informative voting. With more than three experts, other aggregation rules with varying intensities of procedural biases may be investigated. Since any super-majority rule will be procedurally biased, albeit in varying degrees, one may ask if the simple majority rule is the only one where even leakage threats cannot obtain informative voting. However, this will prove to be computationally challenging. Nevertheless, the framework on committees under leakage threat that we propose in this paper should help us to address a number of important aspects of the environment that are currently missing in the literature. For example, an endogenous model of leakage where individual committee members have the option to reveal their personal recommendations (or even the complete vote profile of the committee) to the EV may be considered. A detailed analysis on whether randomness of transparency is welfare enhancing for all possible aggregation rules also remains an open question as is the issue of choice between transparency and opacity under leakage threat that we have highlighted in Remark 1. Another interesting extension would be to drop the assumption that all information sources are equally salient. This may lead to a richer domain of mechanisms where votes are weighted and these weights can be an additional design instrument. Finally, one may allow for private information transmission among experts through deliberation and investigate how leakage affects the value of information flow across experts. We reserve these extensions for future research.

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disclaimer applies.

Appendix

Proof of Theorem 1: We introduce the following definition. A voting strategy σ is called a *cut-off strategy* if there exists a *cut-off talent* $t(x,p;\pi) \in T$ such that σ_i prescribes the following: $m_i = b$ for all $t_i \in T$ if $s_i = b$, and for all $t_i \leq t(x,p;\pi)$ if $s_i = a$ and $m_i = a$ otherwise.

Let $\mu_{w_i} = \Pr[m_i = a|w_i, t_i, \zeta^*]$ where $\zeta^* = \sigma^*$. Let $t^*(x,p;\pi)$ be the cut-off talent in equilibrium. Then $\mu_a = 1 - t^*(x,p;\pi)^2$ and $\mu_b = (1 - t^*(x,p;\pi))^2$. Let $v(\pi, t_i|s_i) := \Pr(w_i = a|\pi, s_i, t_i)$. Let

$$\pi_{w_i}(x, t^*(x,p;\pi)) = \Pr[d_x = A|m_i = w_i, \{w_j\}_{j\neq i}, t^*(x,p;\pi)],$$

and let $\alpha_{d_x}(w_i, \{w_j\}_{j\neq i}, x)$ be the EV's posterior belief that $m_i = a$, given that the EV knows the committee decision d_x and has observed the states in each dimension. Then,

$$\alpha_A(w_i, \{w_j\}_{j\neq i}, x) = \frac{\mu_{w_i}\pi_a(x, t^*(x,p;\pi))}{\mu_{w_i}\pi_a(x, t^*(x,p;\pi)) + (1-\mu_{w_i})\pi_b(x, t^*(x,p;\pi))}, \text{ and}$$

$$\alpha_B(w_i, \{w_j\}_{j\neq i}, x) = \frac{\mu_{w_i}(1-\pi_a(x, t^*(x,p;\pi)))}{\mu_{w_i}(1-\pi_a(x, t^*(x,p;\pi))) + (1-\mu_{w_i})(1-\pi_b(x, t^*(x,p;\pi)))}.$$

Finally let $\alpha(d_x, w_i, x)$ be the probability estimate *held by expert* i of the EV's posterior belief that $m_i = a$. Then,

$$\alpha(d_x; w_i, x) = \sum_{(w_j,w_k)\in\{a,b\}^2} \Pr[(w_j, w_k)|\mathbf{piv}_i, x, t^*(x,p;\pi)]\alpha_{d_x}(w_i, \{w_j\}_{j\neq i}, x),$$

where $\mathbf{piv}_i$ stands for the event that expert i is pivotal. Let $U(m_i;p)$ be the expected payoff of expert i with talent t_i and private signal s_i from the vote m_i. Using the shorthand $v := v(\pi, t_i|s_i)$, we have

$$U(a;p) = p\left[v\tau(a,a,\zeta^*) + (1-v)\,\tau(a,b,\zeta^*)\right] + (1-p)\left[v\left(\alpha(A,a,x)\tau(a,a,\zeta^*)\right.\right.$$
$$\left.\left. +(1-\alpha(A,a,x))\tau(b,a,\zeta^*)\right) + (1-v)\left(\alpha(A,b,x)\tau(a,b,\zeta^*) + (1-\alpha(A,b,x))\tau(b,b,\zeta^*)\right)\right],$$

and

$$U(b;p) = p[v\tau(b,a,\zeta^*) + (1-v)\tau(b,b,\zeta^*)] + (1-p)[v\left(\alpha(B,a,x)\tau(a,a,\zeta^*)\right.$$
$$\left. +(1-\alpha(B,a,x))\tau(b,a,\zeta^*)\right) + (1-v)\left(\alpha(B,b,x)\tau(a,b,\zeta^*) + (1-\alpha(B,b,x))\tau(b,b,\zeta^*)\right)].$$

Lemma 1. *If $s_i = b$, then for any value of t_i, any transparency probability $p \in [0, 1]$ and any voting rule $x \in \{2, 3\}$, we have (i) $v < 1/2$ and (ii) $m_i = b$ is expert i's best response for all talent levels. Hence voting is informative.*

Proof. If $s_i = b$, then expert i with any t_i is strictly better off with $m_i = b$ in any committee $C = (x, p)$ as long as $p \in \{0, 1\}$. [9] Since the expected payoff for an expert in the committee $C = (x, p)$ for any $p \in (0, 1)$ is a convex combination of these payoffs, this proves part (ii). To prove part (i) observe that $v(\pi, t_i|b) < 1/2$ if and only if $1 - t_i < \pi$. But since $t_i \geq 1/2$ and $\pi > 1/2$, this condition must always hold. Parts (i) and (ii) together then imply that in each sub-game with $s_i = b$, voting is informative irrespective of the actual realisation of the random variable t_i. □

Given Lemma 1, it is enough to consider the case when $s_i = a$. Note that in this case, $v(\pi, t_i|a) \leq 1/2$ iff $t_i \leq \pi$. Hence, if expert i is using an informative voting strategy, then it must be true that if $s_i = a$, we have $t(x, p; \pi) = \pi$. It further follows that in equilibrium, $C = (x, p)$ is informative if and only if $t^*(x, p; \pi) = \pi$. Hence for informative committees we have $\mu_a = 1 - \pi^2$ and $\mu_b = (1 - \pi)^2$.

Consider the indifference equation $U(a; p) = U(b; p)$. Let $v^* \equiv v(\pi, \pi|a)$ and $\hat{\tau}(m_i, w_i, \zeta^*)$ be the evaluation when $t^*(x, p; \pi) = \pi$. Note that in this case, $v^* = 1/2$. Also note that by substituting $t^*(x, p; \pi) = \pi$, we are able to express $\hat{\tau}(m_i, w_i, \zeta^*)$ solely in terms of π. Therefore, $U(a, p)$ and $U(b, p)$ are expressible only in terms of π and p. Let $p(\pi)$ solve this indifference equation for the case when $t^*(x, p; \pi) = \pi$. The strategy of the rest of the proof is as follows. We shall show that such a solution never exists when $x = 2$ and exists under certain restrictions when $x = 3$. We proceed as follows.

Suppose $p(\pi)$ exists. Then, $p(\pi) := \frac{N}{N-M}$where

$$\begin{aligned} M &:= v^*(\hat{\tau}(b, a, \zeta^*) - \hat{\tau}(a, a, \zeta^*)) + (1 - v^*)(\hat{\tau}(b, b, \zeta^*) - \hat{\tau}(a, b, \zeta^*)) \text{ and} \\ N &:= v^*(\alpha(A; a, x) - \alpha(B; a, x))(\hat{\tau}(b, a, \zeta^*) - \hat{\tau}(a, a, \zeta^*)) \\ &\quad +(1 - v^*)(\alpha(A; b, x) - \alpha(B; b, x))(\hat{\tau}(b, b, \zeta^*) - \hat{\tau}(a, b, \zeta^*)). \end{aligned}$$

For existence of $p(\pi)$, it is necessary that $p(\pi) \geq 0$. For $p(\pi) \geq 0$, there are two exclusive necessary and sufficient conditions: either (I) $[N \geq 0$ and $N > M]$ or (II) $[N \leq 0$ and $N < M]$. Suppose condition (I) holds. As it is also necessary that $p(\pi) \leq 1$, it then follows that $N \leq N - M$ so that $M \leq 0$. If condition (II) holds, then by similar arguments it must be that $M \geq 0$.

Claim 1. *For all committees $C = (x, p)$ and for all priors $\pi \in (1/2, 1)$, we have $M < 0$.*

[9] This is shown in Lemma 3 of Levy (2004), and Proposition 2 of Levy (2007a).

Proof. Since $v^* = 1/2$, it follows that $M < 0$ if and only if $\hat{\tau}(b, a, \zeta^*) + \hat{\tau}(b, b, \zeta^*) < \hat{\tau}(a, b, \zeta^*) + \hat{\tau}(a, a, \zeta^*)$. In Lemma 2 of Levy (2004), it is shown that $\hat{\tau}(b, b, \zeta^*) < \hat{\tau}(a, a, \zeta^*)$ and $\hat{\tau}(b, a, \zeta^*) < \hat{\tau}(a, b, \zeta^*)$ Hence $M < 0$ always holds. □

Claim 1 rules out condition (II) and for the rest of the proof we shall only consider condition (I). We now give a direct proof of Part (i) of the theorem. Let $x = 2$. It is routine to verify that if $\pi \in \{1/2, 1\}$, then $N < 0$. Next, setting $N = 0$ and solving for π yields a unique real root $\pi \approx -0.2208 \notin [1/2, 1]$. Since N is continuous in π, we conclude that for all $\pi \in [1/2, 1]$ it must be that $N < 0$. Thus we have shown that with $x = 2$, condition (I) can never be satisfied. This proves part (i). We now prove part (ii) of the theorem. Suppose $x = 3$. It is again routine to check the following: at $\pi = 1/2$ we have $N > 0$; at $\pi = 1$ we have $N < 0$; and $N = 0$ if and only if $\pi = \tilde{\pi}(\approx 0.54197)$. Given these, the rest of the proof follows by invoking the fact that N is continuous in π. This completes the proof of the theorem. *QED.*

Proof of Theorem 2: The following lemma will be useful in the proof of Theorem 2.

Lemma 2. *Suppose $t^*(x, p; \pi) \in [1/2, 1]$ solves the indifference equation $U(a; p) = U(b; p)$. Then $t^*(x, p; \pi) > t^*(x, p'; \pi)$ if and only if $p < p'$.*

Proof. By suitable manipulation, the indifference equation $U(a, p) = U(b, p)$ can be expressed as $pL = (1 - p)R$, where $L = U(a; 1) - U(b; 1)$ and $R = U(b, 0) - U(a, 0)$. For $x = \{2, 3\}$, we have: $\frac{\mathrm{d}L}{\mathrm{d}t} > 0$ and $\frac{\mathrm{d}R}{\mathrm{d}t} < 0$.[10] Now consider the cut-off talent level $t^*(x, p; \pi)$ that solves the indifference equation $U(a; p) = U(b; p)$. Fix the value of $t^*(x, p; \pi)$ thus obtained and suppose p rises to p' so that now $p'L > (1 - p')R$. Let $t^*(x, p'; \pi)$ be the new cut-off talent that solves the indifference equation $p'L = (1 - p')R$. The equality $p'L = (1 - p')R$ can be attained in one of the following three ways: either (i) L decreases and R increases, or (ii) both L and R fall but the fall in L is larger than that in R, or (iii) both L and R rise but the rise in L is smaller than that in R. But since $\frac{\mathrm{d}L}{\mathrm{d}t} > 0$ and $\frac{\mathrm{d}R}{\mathrm{d}t} < 0$, cases (ii) and (iii) cannot be true as in each of these two cases the required directions of $t^*(\cdot)$ are opposite to each other. Hence it must be that case (i) holds. But for that case it must be that $t^*(\cdot)$ falls. To complete the proof, we address the special situation where the equation $U(a, p) = U(b, p)$ is solved at $t^*(x, p; \pi) = 1/2$. For that case we can mimic the above proof by considering $p'' < p$ for which $p''L < (1 - p'')R$ and then similarly show that $t^*(x, p''; \pi) > t^*(x, p; \pi)$. □

[10]This is shown in Proposition 1 of Levy (2004) and Proposition 2 of Levy (2007a).

Lemma 2 shows that as the probability of the committee being transparent rises, the cut-off talent t^* falls.[11] Let $x = 3$. Fix p and π and use the shorthand $t := t^*(x, p; \pi)$ for the cut-off talent in equilibrium. Let t^f be the *free* solution from the first order condition of unconstrained optimisation of $W(C(x, p), \pi)$, given by $\frac{\mathrm{d}W}{\mathrm{d}t} = 0$. Then, t_f is given by the following three solutions: $t_1^f = \frac{1}{2\pi-1}$, $t_2^f = \frac{A+10\pi^2-7\pi+3}{6(2\pi-1)}$, and $t_3^f = \frac{A-10\pi^2+7\pi-3}{6(1-2\pi)}$, where $A = (100\pi^2 - 36\pi + 9)^{1/2}|\pi - 1|$.

To prove Part (i) observe that since $0 < t < 1$, it suffices to check if there exists π such that $t = \pi$ from any of the above solutions. Inserting $t_k^f = \pi, k = 1, 2, 3$, it is routine to check that for the first two solutions, viz. t_1^f and t_2^f, this implies $\pi = 1$. For the remaining solution t_3^f, we have $\pi = 1$ or $\pi = 0$. But since $\pi \in (1/2, 1)$, this proves that informative voting cannot be a solution to the aggregate welfare maximisation problem.

To prove Part (ii.a) define the function $S(t) = 4t^3 - 9t^2 + 6t - 1$. Note that $\frac{\mathrm{d}W}{\mathrm{d}t}|_{\pi=1/2} = -3S(t)$. We first show that $S(t) > 0$ for all $t \in [1/2, 1]$. For that, by invoking the fact that $S(t)$ is a continuous function, it is sufficient to observe that $S'(t) = 0$ has exactly two solutions, $t = 1/2$ and $t = 1$, $S(1) = 0$ and $S(1/2) = 0.25 > 0$. It then follows that $\frac{\mathrm{d}W}{\mathrm{d}t}|_{\pi=1/2} < 0$ for all values of t. By continuity of $W(\cdot)$ in π, it follows that there exists $1/2 < \pi^* < 1$ such that for each $\pi \leq \pi^*$, we have $\frac{\mathrm{d}W}{\mathrm{d}t} < 0$ for all values of t. Thus for such values of π, it must be that $t = 1/2$. Now recall the definition of $t^*(3, p; \pi)$ that solves the indifference equation $U(a; p) = U(b; p)$. By Lemma 2, it follows that for each $\pi \leq \pi^*$, $p = 1$ uniquely maximises $W(3, p, \pi)$.

We prove Part (ii.b) in a similar fashion. Define the function $H(t) = t^5 - 5t^4 + 10t^3 - 10t^2 + 5t - 1$. Note that $\frac{\mathrm{d}W}{\mathrm{d}t}|_{\pi=1} = -18H(t)$. We first show that $H(t) < 0$ for all $t \in [1/2, 1)$. This is established by the following facts that can be easily checked: $H'(t) = 0$ has a unique real root $t = 1$, $H'(1/2) = 5/16 > 0$, $H(1/2) = -1/32 < 0$ and $H(1) = 0$. Hence by continuity of $H(\cdot)$, it follows that $H(t) < 0$ for all $t \in [1/2, 1)$. By continuity of $W(\cdot)$ in π, it follows that there exists $\pi^{**} < 1$ such that for each $\pi \geq \pi^{**}$, we have $\frac{\mathrm{d}W}{\mathrm{d}t} > 0$ for all values of $t \in [1/2, 1)$. By Lemma 2 it follows that for each $\pi \geq \pi^{**}$, the value of p which maximises $W(3, p, \pi)$ is $p = 0$.

We now prove part (iii). Given what we have proved thus far, it would suffice to show that there exists a threshold value $1/2 < \hat{\pi} < 1$ such that for all $\pi < \hat{\pi}$, we have $W(2, 1, \pi) > W(3, 1, \pi)$. We first prove the following claim.

Claim 2. *When $p = 1$ and $\pi = 1/2$, then $t^*(x, p; \pi) = 1/2$.*

[11] The proof of the lemma uses a 'convexification' argument on two extreme committees, namely, $C(x, 0)$ with weight $1-p$ and $C(x, 1)$ with weight p. However it is important to note that the cut-off talent $t^*(x, p; \pi)$ is not in general equal to $pt^*(x, 1; \pi) + (1 - p)t^*(x, 0; \pi)$.

Proof. Recall the indifference equation $pL = (1-p)R$ used in the proof of Lemma 2. At $p = 1$ this indifference equation is solved if and only if $L = 0$, where $L = U(a;1) - U(b;1)$. At $t^*(x,p;\pi) = 1/2$, we have $\tau(a,a,\zeta^*) = \tau(b,b,\zeta^*)$ and $\tau(b,a,\zeta^*) = \tau(a,b,\zeta^*)$. Using these, one can obtain $L = (1 - 2v)\left(\tau(a,a,\zeta^*) - \tau(b,a,\zeta^*)\right)$. From Lemma 2 in Levy (2004) it follows that $\tau(a,a,\zeta^*) > \tau(b,a,\zeta^*)$. Hence $L = 0$ if and only if that $v = 1/2$ which is possible at $\pi = 1/2$ if and only if $t^*(x,1;1/2) = 1/2$. □

Let $x = 2$. Using Claim 2 we have $W(2,1,1/2) = \frac{15}{8}$. From part (2) we know that when $\pi < \pi^*$ (where π^* is as defined there), aggregate welfare under $x = 3$ is maximised if and only if $p = 1$. So consider $\pi = 1/2$, $p = 1$ and $x = 3$. Using Claim 2 once more we have $W(3,1,1/2) = \frac{27}{6}$. It follows that $W(2,1,1/2) > W(3,1,1/2)$. Note further that $\frac{\mathrm{d}W(x,p,\pi)}{\mathrm{d}t}|_{x=3,\pi=t=1/2} = -\frac{3}{4} < 0$, $\frac{\mathrm{d}W(x,p,\pi)}{\mathrm{d}t}|_{x=2,\pi=t=1/2} = 0$ and $\frac{\mathrm{d}^2W(x,p,\pi)}{\mathrm{d}t^2}|_{x=2,\pi=t=1/2} = -6 < 0$. Theorem 1 in Levy (2004) shows that $\frac{\mathrm{d}t(\cdot)}{\mathrm{d}\pi} > 0$ for all $x \in \{2,3\}$ when $p = 1$. Hence, as π rises from $\pi = 1/2$, the cut-off talent t rises for both $x = 2$ and $x = 3$. By continuity of $W(\cdot)$ in π and t, there exists $\hat{\pi} > 1/2$ such that $W(2,1,\pi) > W(3,1,\pi)$ for all $\pi < \hat{\pi}$. This completes the proof of Theorem 2. *QED.*

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