Strategic dissent in the Hotelling-Downs model with sequential entry and private information*

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October 22, 2014

Abstract

We analyze the Hotelling-Downs model of winner-take-all elections with sequential entry where $n \geq 2$ 'office-seeking' candidates with privately known qualities choose entry decisions and commit to policy platforms on entering. Voters receive informative public signals about the quality of each contestant once all platforms are announced. We first characterize two-party equilibria when the order of entry is exogenously given. In these equilibria, entry can occur in any 'round' with positive probability: high-quality candidates signal their type through showing ideological dissent with the voters while low-quality ones randomize between (mis)-signaling quality through dissent and staying out. Interestingly, while informative public signals can keep low-quality candidates out of competition up to a certain degree, electoral competition improves the voter's information about candidate types beyond what the signals can reveal. However this endogenous mechanism of strategic information transmission leads to political polarization. We then endogenize the order of entry to show that high quality candidates either enter early or late while all low quality candidates either stay out or enter late. Moreover, while extremism continues to signal quality, there must be a gradual moderation in ideology although information revelation is non-monotonic in time with

*We thank Sandro Brusco, Antonio Cabrales, Colin Rowat and seminar participants at the University of Birmingham and the Indian Institute of Management - Calcutta for insightful comments and Katharine Inglis for research assistance .

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full revelation for early and late entrants and only partial revelation for intermediate entrants.

Keywords: Sequential entry, Unobserved quality, Strategic dissent, Polarization, Endogenous Order.
JEL Classifiers: C72, D72, D82

1 Introduction

Models of electoral competition typically analyze the type of policies that arise in equilibrium and pay less attention to other dimensions that may influence voters. Yet media coverage of elections suggest that policy is only one dimension of what voters take into account and non-policy issues are often predominant in deciding electoral outcomes. This non-policy dimension, recurrently described as firmness of purpose (or character) or quality of governance, is something that in principle voters agree as desirable and which can persuade them to vote for a candidate even when they disagree with his policies. This is what Stokes (1963) terms as valence. But when this non-policy dimension is private information to the candidates, not pandering to the wishes of electorally pivotal voters can itself be regarded as a signal of strength. A large body of literature has developed that looks at what is called the marginality hypothesis which suggests that weaker candidates are more likely to contest with electorally popular platforms.[1] But beliefs that pandering is symptomatic of low quality may of course lead to strategic choices by politicians to deliberately distance themselves from popular ideologies - we call this *strategic dissent.*

There is evidence of ideologically unpopular politicians (or parties) winning elections because voters believed they would be more efficient or trustworthy in what they do, making up for any loss in ideological alignment. Margaret Thatcher may have been the most conservative and certainly the most radical Prime Minster that Britain had (in the words of her biographer Charles Moore) [2] but she won elections and an IPSOS Mori Poll in 2011 finds that she is considered the most capable Prime Minster in the last few decades[3]. To take another example, it is believed that the staunch left-wing politician Paul Wellstone was seen by the Minnesota voters as having integrity and although his opponent Rudy Boschwitz's ideological position was popular, starting as a clear underdog Wellstone surprised all with a remarkable victory in the 1990 US Senate elections. In 1999 the Dutch party VLD (Open Vlaamse Liberalen en Democraten) won on a right wing platform (contrary to when they deliberately chose a policy aligned with the majority and lost in 1995).[4] A recent example

[1] Although Fiorina (1973) offers some evidence to the contrary, there is certainly some evidence of the marginality hypothesis (see Ansolabehere et. al, 2001 and Griffin, 2006 for recent empirical support for the hypothesis). In this resepct, Bernhardt et. al (2011) provide a theoretical explanation for the mixed empirical results on valence and extremism in a model of repeated elections with ideologically driven politicians.

[2] http://www.telegraph.co.uk/news/politics/margaret-thatcher/10005886/ Radical-egotistical-romantic-innocent-the-real-Margaret-Thatcher.html

[3] See http://www.itv.com/news/update/2013-04-09/thatcher-remembered-as-most-capable-prime-minister/

[4] See http://en.wikipedia.org/wiki/Open_Vlaamse_Liberalen_en_Democraten.

of a politician who seems to deliberately flaunt a certain amount of sectarian and economic extremism is India's current Prime Minister Narendra Modi, who led his party (the BJP) to power in the recent parliamentary elections. Several press releases and opinion polls suggest that voters saw him as an efficient and decisive leader which compensated for his extreme image. Indeed as the New York Times reports, Modi, 'has emerged with a bold, right-wing narrative in a country with a staunchly socialist past' even while the centrist Congress is struggling with an image of policy paralysis. [5]

These examples seem to indicate that ideological extremism can be used to signal quality and our paper analyzes conditions under which strategic extremism occurs in a Hotelling-Downs (HD) model (Hotelling (1929); Downs (1957)) with one-dimensional policy space, free entry and incomplete information about candidate quality. The choice of a Downsian framework (or purely office-seeking candidates) is to allow us to filter out the impact that party ideology may play in the choice of platforms. As candidates do not care for policy in the classical HD world, any deviation from the median voter's ideology must come from strategic reasons. The model we study has $n \geq 2$ potential entrants (or candidates) and a decisive voter group (which we may think of as the median voter).[6] Free entry puts pressure on parties to move towards popular ideologies in order to thwart future entry. Thus, while it is a stark way to model endogenous entry, obtaining extremism in such a framework if anything understates the forces for policy divergence. Unlike most of the literature on valence which studies competition between two given parties, we endogenize the size of political participation under the free-entry assumption. A complication that arises is that HD models with free entry run into problems concerning equilibrium existence. While competition between two parties yields a unique Nash equilibrium outcome where both parties locate at the ideal policy of the median voter (often called the Median Voter Theorem that has remained central to the formal literature on elections) when there are $n > 2$ potential candidates a Nash equilibrium in pure strategies fails to exist (see Osborne 1995). Further with sequential entry, which is the focus of this paper, equilibrium characterization becomes a more intriguing problem. While for $n = 3, 4$ there exists a unique sub-game perfect equilibrium where only candidates 1 and n contest with the median policy (while all other candidates stay OUT), this result remains an open conjecture for $n \geq 5$.[7] Moreover, as we also study the case when the order of entry is determined endogenously as an 'equilibrium' outcome, our model faces the challenges of free entry from both simultaneous and sequential entry frameworks. Given this, we ask if incomplete information can ease the existence problem and allow one to analyze equilibrium behavior with free entry.

Following the HD framework, we assume that entrants can credibly commit to any policy

[5] It is striking that he has support even among the Muslim minority towards whom he is supposed to be insensitive and he has not always pandered to the Hindu majority in his actions either (see http://world.time.com/2012/03/16/why-narendra-modi-is-indias-most-loved-and-loathed-politician/).

[6] Our results for the case where the order of entry is exogenously given is generalizable to the case when there is a distribution of voter's ideal policies (c/f. Remark 2).

[7] see Osborne's website at http://www.economics.utoronto.ca/osborne/research/CONJECT. for the conjecture with a finite number of candidates. Of course, the conjecture is not applicable when $n = \infty$, a case also studied by us.

and policies are perfectly observable. However each candidate has a non-policy quality (or valence) parameter which is not known to the voter or other candidates. Voters receive partially informative public signals about the quality of the final contestants after they have made their ideological commitments (whether in an exogenous or endogenous order of play). One can think of this as occurring because of media investigation or simply from the party campaigns that reveal some information about the candidate through her speeches, handling of questions etc. Given the signal and the announced policies of the candidates, the voter makes a choice in a winner-take-all election. As in the environment where the Osborne conjecture is analyzed, we assume that while the cost of entering the contest is zero (free entry), candidates incur a cost only if they do not tie for the first place.[8]

1.1 Results

Fixed order of entry: With an exogenously fixed order of entry, we prove generic existence of equilibria that exhibit the phenomenon of strategic dissent and is consistent with Duverger's Law of two-party systems (see Duverger (1964)). We show that for each $n \geq 2$ there are conditions (on costs and signal precision) under which exactly two candidates choose to enter the contest while all other candidates stay out. Entry can take place in any period and unless the last candidate (called n) faces a history where there is no past entrant (the only case where policy becomes arbitrary), each contesting party commits to policy platforms that are away from the median voter's ideal point, thereby leading to political polarization (with probability 1 for the case $n \geq 3$)[9]. While a high quality entrant contests with a dissenting platform with probability 1, a low quality party randomizes between contesting with an equally dissenting platform and staying out. As a consequence, ideological dissent becomes a signal of quality although the equilibrium is only partially revealing. As expected, the more weight the voters attach to the quality parameter the bigger is the deviation of the winner's policy from the median voter's ideal point. We show that to obtain this result one requires relatively high but bounded costs and intermediate degree of exogenous signal precision. We also show that for each finite n, the median voter (prior to receiving exogenous signals) strictly prefers the first entry to take place as early as possible in the entry game; but once the first entry takes place he is indifferent about the timing of the second entrant. Moreover, he always strictly prefers the second entrant to the first. This implies that in equilibrium the earlier the first entry more likely it is to be of high quality but the second entrant is always more likely to be high quality than its first-entrant counterpart.[10] We then show that as n grows unboundedly, this strict temporal and history-dependent preference of the voter disappears so that in the limit neither timing nor position of the entrants matter. A consequence of this limit observation is that the probability of a voter-pandering contestant

[8]It is worth noting that the main results of our model go through even with a standard entry cost formulation where the cost is paid at the time of entry and borne irrespective of the electoral outcome that follows.

[9]With $n = 2$ one can have policy convergence but equilibria with extremism still exists.

[10]When we endogenize the order of entry we find that this temporal aspect of the preference ordering gets reversed.

decreases as the number of potential competitors increases even though two-party contest is maintained. This is counter intuitive as one expects stronger centripetal forces with more competition. Our analysis also proves that this limit equilibrium is indeed an equilibrium for the case when $n = \infty$. It is important to observe in the background of these results that they continue to hold with generic distributions of voters' ideal points as long as these distributions are sufficiently thin over extreme policies.

The limit equilibrium (when $n = \infty$) where the voter is indifferent between the two contesting parties has some interesting comparative static properties. Starting from a certain level of informativeness of exogenous signals, an increase in informativeness has two effects: extremism falls, which improves voter welfare but it comes at a cost as the low type's probability of entry increases reducing voter welfare. Given this tradeoff one may ask the following: can better public information sources hurt voters? We show that fortunately not, that is, the voter's ex-ante welfare must increase with more informative signals. Finally, we show that even if the prior probability of high quality candidates in the population becomes very small so that incomplete information is almost absent in the environment, the two party equilibrium with platform extremism continues to exist. This result stands out as an interesting contrast with Osborne's conjecture though the two models are not conceptually comparable.

Given the extensive literature on HD models with 2 candidates, we look at other possible equilibria when $n = 2$ with a fixed order of entry. We show that dissent is not necessary to signal strength as a mere entry (even with a voter-pandering policy) can serve this purpose as well. In particular, there are indeed equilibria where the high type entrant contests under a popular platform while a low type randomizes between that and staying out. Yet, under a plausible assumption on signals such an equilibrium with full pandering becomes fragile. The assumption we make is that the more extreme a candidate's position, the more likely it is for him to generate public signals. This is plausible as there is strong evidence that the press investigates extreme candidates more routinely (see for example McCluskey and Kim, 2012). Such an assumption makes higher quality candidates deliberately choose dissent, thereby increasing the probability of getting favorable signals, leaving low quality rivals no other option but to randomize between staying out and mimicking high quality actions of unpopular platforms. Further, with free entry ($n > 2$), an equilibrium where two parties stand at the median voter's ideal point will be fragile and particularly so when we move to general distributions of voter's ideologies.

Endogenous order of entry: Finally we look at the case where parties are allowed to choose when to enter the contest.[11] Endogenous timing leads to the possibility of simultaneous entry by more than two candidates in equilibrium and for this reason, the results we obtain

[11]To the best of our knowledge there is a single existing work (see Osborne (2000)) on electoral competition where candidates can choose when to enter. In that model it is assumed that parties are uncertain about the location of the median voter but there is no valence parameter for the parties. Osborne shows that with three candidates there exists an equilibrium in which two candidates enter simultaneously at distinct positions in the first period while either the third candidate stays out or enters at a platform between the first two.

here are not generalizable to arbitrary distributions of voter's ideal policies as then we enter non-existence of pure strategy equilibria in certain sub-games.

Where candidates are free to choose when to announce their candidacy, time can be an additional dimension that parties may potentially use to signal quality. An important and novel question to be asked here is whether the time of entry itself can act as an instrument to signal private strengths and whether this enhances or dampens the political extremism that we have shown to be consistent with equilibrium behavior with a fixed order of play. For example, is it more attractive for good quality parties to signal strength by announcing policy stands early in the electoral process rather than using ideological extremism to do the same or is it that even an early entry option is not enough to lead to ideological moderation. Interestingly we find a somewhat surprising outcome that can be obtained in equilibrium. In line with our intuition, early entry is necessarily from high quality parties while late entries are either from those who reveal themselves as low quality or there is pooling across types so that while entry is still informative beyond what voters believe a priori, full revelation is not possible.[12] However, early entrants must necessarily be ideologically more extreme than their rivals who arrive late in the contest. This feature is not a necessity when the order of entry is given exogenously.

1.2 Related literature

Our results contrast with Groseclose (2001), who finds conditions under which marginal candidates (that is those with low quality) take more extreme positions unlike in our case where this is never true. Groseclose obtains this in a model where quality is perfectly observable but the voter preferences are not perfectly known. Given this, the weaker (in valence) candidate's only hope is not to be near the stronger candidate on the policy line since if the pivotal voter (whose exact position is unknown) sees two candidates close to each other, she will vote for the one with higher valence. The idea that platform choice can affect voters' beliefs about an unobserved but important trait of a party has been analyzed in Kartik and McAfee (2007). Similar to our work, they show how parties indulge in strategic dissent (thus choosing policies away form the median). While they study two candidate games (we look at endogenous entry and obtain two party contests as equilibrium outcomes) Kartik and McAfee assume the (exogenous) existence of non-strategic candidates with character (the so called 'crazy types' as in Kreps et al. (1982)) who act according to their beliefs about what would be the 'right' policy (modeled as a random process that assigns probabilities to different policy platforms), rather than catering to popular demands. Voters like character and since strategic office-seeking candidates typically announce popular policies, extremism attracts favorable attention. Given this, strategic candidates cannot afford to be too populist anymore, although on average they are closer to the median voter than an expected crazy type. In their model, candidates with character are essentially non-actors and have no

[12] In the very special case when there is actually no high quality candidate in the population, the equilibrium path of play can select the outcome with strictly positive probability where all low quality candidates enter at the voter pandering policy platform and get revealed.

explicit desire to signal anything to anyone. In contrast, we endogenize participation of different candidate types and their policy choices. Moreover, the notion of character we use is more about productive efficiency in the political arena (like good governance rather than appropriateness of the policy in question since in our model, voters are fully informed about policy appropriateness) that is signaled by choice of unpopular policies.[13] There are also some other papers which show that some form of extremism in actions signals quality. Starting with Rogoff (1990) who looks at higher than optimal deficit spending, a large body of literature has sprung up where politicians take more extreme positions than socially optimal to signal quality. Applications include inefficient transfers to special interest groups (Coate and Morris (1995)) and excessive litigation in the courtroom (Bandyopadhyay and McCannon (2013)).[14] However, in all these models policy makers care for policy as well as winning and thus signaling is credible in such models because it is directly costly for them to choose an ideology more extreme than their own favorite policy about which voters are fully informed.

The rest of the paper is structured as follows. In Section 2 we describe the model where the order of entry is fixed. Our main results with a fixed order are in Sections 3 and 4 where in Section 4.4 we also introduce some modifications to the benchmark model to allow for extremism driven public signals. Section 5 develops the model for endogenous entry and discusses the main result for the new framework. We conclude in Section 6 where we also summarize our main results. All proofs are in an appendix.

2 Model

A politically decisive constituency with ideal policy $m \in \mathbb{R}$ and Euclidean preference over the policy line $\mathbb{R}$ selects a candidate via winner-take-all elections. There are $n \geq 2$ candidates called $i = 1, 2, \ldots, n$ who arrive in an *exogenous order* to decide whether to stay OUT or contest the elections by committing to a platform in $\mathbb{R}$. We denote by x_i the platform commitment of candidate i if he chooses to contest. The distance $z_i = |x_i - m|$ is the extent of dissent of platform x_i with respect to the ideal policy of the decisive constituency.

Each candidate i is endowed with a privately known quality parameter θ_i which can either be high ($\theta_i = H$) or low ($\theta_i = L$). We assume that quality parameters across candidates are stochastically identical and independent and denote by γ the prior probability that $\theta_i = H$. Each candidate is fully informed about the history of past actions so that a strategy for

[13] Carrillo and Castanheira (2008) also obtains strategic extremism although in their model quality can be enhanced through unobservable investment unlike in our case or that of Kartik and McAfee where it is treated as an immutable candidate endowment. There are other models of electoral competition where anti-pandering is obtained. Kartik et.al (2012) analyze a classical Hotelling-Downs model where politicians have better information about the state of the world and it is shown that in equilibrium, anti-pandering will be seen i.e. positions relatively extreme to the median voter will be taken. Honryo (2013) obtains extremism as a result of candidates trying to signal their competence about their ability to read the true state of the world.

[14] See Besley (2006) for a good survey of the literature.

a candidate is a mapping from his type (H or L) and the game's history to a probability distribution over the action set $\mathbb{R} \cup \{\text{OUT}\}$. We denote an individual strategy by σ and a strategy profile by σ^n. Once all candidates have made their choices, voters form (interim) beliefs about their qualities. Let $\rho_k(\sigma^n)$ be this interim belief that contestant k is of quality H at strategy profile σ^n.

Once all candidates have made their choices, the constituency receives exogenous public signals (for example, from the press) about the quality of each contestant. Denote by s_k the signal generated by contestant k with $s_k \in \mathcal{S} \subset \mathbb{R}$. The probability of obtaining a signal $s_k \in \mathcal{S}$ conditional on the true realization of θ_k is denoted by $\phi(s_k|\theta_k)$. We assume that $\phi(\cdot|\cdot)$ satisfies the monotone likelihood ratio property so that while public signals provide only partial information, a higher signal value indicates a higher probability of the candidate being of high quality. After obtaining the public signals for each contestant, the voters form (posterior) beliefs about the quality of each contestant. We denote this posterior by $\rho_k(\sigma^n|s_k)$ which is simply a Bayesian update of $\rho_k(\sigma^n)$ given the prior γ and the function $\phi(s_k|\theta_k)$.

Voters are expected utility maximizers with preferences being linear in quality. In particular, let $h > 0$ be the utility from electing a contestant of quality H while it is 0 when the contestant is of quality L. Thus, the voter's payoff from electing contestant k at strategy profile σ^n who announces platform x_k and reveals a public signal s_k is

$$-|x_k - m| + \rho_k(\sigma^n|s_k)h. \tag{1}$$

In the absence of a unique maximizer, voters randomize (equiprobably) over the set of maximizers so that if there are $w \geq 1$ such maximizers, then each maximizing contestant wins with probability $1/w$ (while those not in the maximizing set lose with certainty). Each candidate obtains a payoff of 0 if he stays OUT, a payoff of $1/w$ if he is among the w contestants who tie for the first place and $-c$ otherwise, where $0 < c \leq 1$.[15] Candidates are also expected utility maximizers and hence enter the contest if and only if their expected payoff is non-negative.

The above environment leads to an extensive form game of incomplete information where we study sequential equilibria. An equilibrium will be called informative if interim beliefs $\rho_k(\sigma^n)$ at equilibrium outcomes (once platform commitments are announced but before exogenous signals are released) are different from the prior γ. It is easy to see that standard cheap talk messages about one's own quality will fail to transmit any information in this environment and information transmission (if any) must be through some costly signaling device. In light of Duverger's Law of two party systems, in what follows we will mainly study characteristics and existence of equilibria that lead to exactly two contesting parties.

[15]Since the total surplus in the political market is 1 costs higher than 1 would seem unreasonable.

3 Informative equilibria with $n = \infty$

We begin with the following remark that characterizes equilibrium behavior under full information.

Remark 1 (Full information:). *In the full information version of our model there is a unique subgame-perfect equilibrium where all high quality candidates contest by committing to the platform m while all low quality candidates stay OUT. On the other hand, for a general distribution of voter's ideal policies over the policy line $\mathbb{R}$ and where all candidates are identical in quality, we are back to the Osborne conjecture. In Section 4.2 we will address the case when the amount of incomplete information is arbitrarily close to zero in a particular way.*

We first consider the case where there are a potentially infinite number of candidates (i.e., $n = \infty$).[16] Central to the construction of an informative equilibrium in this paper is that candidates cannot afford to show ideological dissent with an all-important group of voters unless they are reasonably confident that exogenous signals about their quality will be favorable. When voters understand this, beliefs about quality may depend strongly on platform commitments, favoring contestants who commit to platforms away from m. However, this opens up opportunities for low quality candidates to mimic platform choices of high quality candidates. As beliefs of the voters are Bayes consistent on the equilibrium path, the equilibrium we propose can at most be partially revealing in the following sense: whenever a high quality candidate enters, he does so by committing to platforms away from m while low quality candidates randomize between dissent and OUT. In the construction of the equilibrium, voter's beliefs satisfy a weak-monotonicity requirement (see Appendix for the formal definition) which implies the following: there exits a cutoff dissent $z > 0$ such that any contestant is believed to be of high quality with positive probability if and only if his platform commitment is outside the interval $(m-z, m+z)$. In the conclusion we discuss why working with these beliefs is not unduly restrictive for preserving the qualitative features of our model. But most importantly, we show (in Remark 2) that equilibria consistent with weakly-monotone beliefs remain robust to a model where instead of have a decisive constituency with common ideal policy there is a general distribution of voters' ideal policies provided at least %50 of the mass of voters lie within the interval $[m - z, m + z]$.

Our first result is Proposition 1.

Proposition 1. *With infinitely many candidates and for intermediate cost c and precision of exogenous signals, there exists an equilibrium where with probability 1 exactly two candidates contest. Entry can take place in any period where, for some $z > 0$, one contestant commits to the platform $m + z$ while the other to platform $m - z$; while a high quality entrant does so with probability 1, a low quality entrant does so with a probability p with $0 < p < 1$ and otherwise stays OUT of the contest. As a consequence, platforms reveal information and the probability of obtaining the voter pandering platform m in equilibrium is zero.*

[16]One can think of this as a pre-election time period where each point in time corresponds to a candidate. To be sure, we also note that this is a case where the Osborne conjecture is not applicable.

The equilibrium has a number of features that we discuss here (details are found in the appendix). First, no candidate contests by pandering to the all-important constituency. Second, although ideological dissent does not reveal high quality with certainty, it is indicative (with respect to the prior γ) of that. Hence, ideological dissent indeed signals strength and elections succeed in restricting low quality participation to some extent. In contrast with Kartik and McAfee (2007) this is obtained without relying upon the existence of crazy types in the population of politicians. Third, entry can appear in any period although the probability of late entry is small. These features contrast sharply with full information case discussed in Remark 1. While Proposition 1 is based upon the existence of an all-important constituency, it can be verified that if instead one had a distribution of voter's ideal policies then the equilibrium would continue to hold as long as the mass of voters with ideal points in the policy interval $[m-z, m+z]$ was at least half of the total population of voters, so that the size of equilibrium dissent is relatively large. If h (the preference intensity for quality) is large, the requirement is indeed weak.

Existence of the equilibrium requires some 'tightness' in the environment both in terms of a sufficiently high cost c and a sufficiently high (but imperfect) signal strengths (as highlighted in last paragraph of the proof in the appendix). The strategy profile (see appendix for a full description) that sustains this equilibrium requires a low quality candidate to randomize between dissent and OUT at two different types of histories: (i) on the equilibrium path of play with history of no past entry or exactly one existing entrant at either $m-z$ or $m+z$ and (ii) off-the-equilibrium path of play where at least one of the two platforms in the set $\{m-z, m+z\}$ remains unoccupied but (possibly many) other platforms are taken. Since randomizing between OUT and a platform in $\{m-z, m+z\}$ entails an expected payoff from entry to be equal to zero for a low quality candidate, when this randomization probability is fixed, it requires expected payoff from entry in $\{m-z, m+z\}$ to be same across several possible histories. However, we prove that it is enough to look at two particular histories: one where there is only one entrant with platform $m+z$ and the other where there are exactly two entrants one at $m+z$ and the other at m. Claim 1 in the appendix goes further to show the following. Suppose the randomization probability and the size of dissent is such that prior to receiving signals, the constituency is indifferent between a contestant at m (who is believed to be of type L with probability 1) and a contestant at $m+z$ (or $m-z$) so that post-platform signal endorsements decisively tilt the voter's preference towards one of these contestants. Then it is sufficient to consider only the history on the equilibrium path of play. This in turn implies that an equilibrium exists where voters ex-ante welfare does not depend on the point in time when there is political entry. As we shall see, this pre-signal indifference is destroyed when there are a finite number of candidates (see Section 4).

While the requirement of pre-signal indifference is not necessary, equilibria that respect this indifference condition allows us to make some straightforward comparative static analysis with respect to the informativeness of post-platform signals. In this regard, we obtain a somewhat surprising result that while better post-platform signals reduce extremism they also reduce the expected quality of contesting parties. To see this, consider the equilibrium path of play and note that irrespective of the level of signal precision, if we are in the parameter range where the equilibrium exists, it must be that the expected payoff of the low quality entrant remains fixed at 0. This condition (see (5) in the appendix) implies two things. First as the signal precision increases, the expected payoff $\Pi(L, H|\sigma^{\infty})$ of type L entrant at $m + z$, conditional on his opponent at $m - z$ being of type H falls, while the expected payoff $\Pi(L, L|\sigma^{\infty})$ of type L entrant at $m + z$, conditional on his opponent at $m - z$ being of type L remains fixed at $\frac{1-c}{2}$. This must imply that the equilibrium pre-signal belief $\rho = \frac{\gamma}{\gamma + p(1-\gamma)}$ that a candidate with dissent is of high quality must fall to maintain this indifference condition since existence of this equilibrium requires $\Pi(L, H|\sigma^{\infty})$ to be negative. At the same time since the voters remain indifferent between a contestant at $m+z$ (or $m-z$) and a contestant at m, it follows (see Claim 1) that $\rho h = z$. This would imply that the degree of dissent z must fall. Hence, increased precision of exogenous signal has two opposing impacts. On the one hand high quality participation decreases (which is bad for the voters) while on the other political extremism decreases as well (which is good for the voters). But by construction, the voters derive the same pre-signal utility due to the requirement of pre-signal indifference. However this implies that higher signal precision must benefit the voters and post-signal beliefs will be more informed. That is, the gain from policy moderation over compensates for the loss from reduced expected quality of political representation. In other words, a better press for example can lead to populism and inefficient governance. We collect this in Corollary 1.

Corollary 1 (Local comparative statics)**.** *: As long as the equilibrium in Proposition 1 continues to exist where the voters enjoy pre-signal indifference between dissenting and pandering contestants, an increase in signal precision leads to policy moderation but higher incidence of low quality political representation. However ex-ante voter welfare increases unambiguously.*

The following result is important and valid for our results with an exogenous order of entry.

Remark 2 (On general distribution of voters' ideal policies:)**.** *WM beliefs have an important implication on the profitability of policy platforms that makes the equilibrium reported in Proposition 1 reasonably robust even under a general distribution of voters' ideal points. To see this, let F be this general distribution of voters' ideal points on the ideology/policy space $\mathbb{R}$ with density f such that now m is the median voter's ideal policy. Let $z > 0$ be the cutoff level of dissent in the WM beliefs. We first make the following observation:* Suppose the median voter with ideal policy m weakly prefers a candidate at some policy in $\{m-z, m+z\}$ to a candidate at m (who has revealed himself to be of type L). Then a voter with ideal policy $y \in (m-z, m+z) \setminus \{m\}$ strictly prefers one of the candidates in the set $\{m-z, m+z\}$ to any candidate $y' \in (m - z, m + z) \setminus \{m\}$. *To see this, take some voter with ideal policy $y \in (m, m + z)$ and notice that if $y' = y$ then voter y strictly prefers the candidate at $m + z$ to y' since by virtue of the WM beliefs, y' is assumed to be of type L. Thus the voter with ideal point y will strictly prefer the candidate at $m + z$ to $y' \neq y$ as well. The argument is symmetric on the left side of m where these voters will strictly prefer $m - z$ to any candidate $y' \in (m-z, m+z)$. This implies that no entry within the interval $(m-z, m+z)$ will ever be* *profitable. Next note that in Proposition 1 we have considered an equilibrium where the degree of dissent z is symmetric on both sides of the median m.*[17] *So suppose $F(\cdot)$ is symmetric over the interval $[m-z, m+z]$, but not necessarily symmetric over the entire policy line $\mathbb{R}$. Then it follows that $F(m-z) = 1 - F(m+z)$ so that if $F(m-z) < \frac{1}{2}(F(m+z) - F(m-z))$, then entry on the flanks (that is to the left of $m-z$ and to the right of $m+z$) cannot be profitable. This condition reduces to $F(m-z) < \frac{1}{3}F(m+z)$.*[18] *To take a more concrete example, suppose the policy line is the unit interval $[0,1]$ and $F(\cdot)$ is uniform over this entire interval. This yields $z > 1/4$, a condition that can be easily satisfied if h is large enough (since $z = \rho h$ in equilibrium), that is, the mass of voters in the interval $[m-z, m+z]$ is $> 50\%$. We note that this is not a non-generic restriction on $F(\cdot)$.*

Can there be a two-party equilibrium that is partially revealing and yet obtains the voter pandering platform m with positive probability? Consider the following strategy profile: on the equilibrium path of play, candidates stay OUT if the platforms m and $m+z$ (or $m-z$) are already occupied; otherwise a low quality candidate randomizes between dissent (with a vacant policy in the set $\{m-z, m+z\}$) and m while high quality candidate takes a vacant policy in $\{m-z, m+z\}$ with probability 1; moreover beliefs follow weak monotonicity property at the equilibrium dissent z. Note first that all entry takes place in the first two periods so that for characterization of this equilibrium the number of candidates is irrelevant although this number has a strong impact on existence of the equilibrium under study. In particular, it will be less robust to future entry than the one proposed in Proposition 1 when we allow for general voter distributions, requiring very heavy concentration of mass around the ideology m to block entry on the flanks that can now be close to m. For these reasons we believe such an equilibrium is less important in the environment under study.[19] Nevertheless, existence of such an equilibrium will require that the voters strictly prefer a candidate at m over a dissenting candidate prior to receiving exogenous signals. This can be readily understood from that following observation. If candidate 2 observes that candidate 1 has entered at m (an outcome that now appears on the equilibrium path of play with strictly positive probability), he obtains a payoff of $1/2$ from contesting at m and a payoff of $\mu - c(1-\mu)$ from standing at $m-z$ where μ denotes the probability that he wins in this

[17] This is not necessary and if for example the dissent on one side was smaller, we would need ρ to depend accordingly (being lower for the policy with smaller dissent so that the politically decisive constituency remains indifferent between the two candidates). This is a complication we will avoid throughout the paper to keep the analysis simple.

[18] To be sure, if the degrees of dissent were not symmetric about the median m (as indicated in footnote 17) this condition can be modified in the following fashion: if z_L is the left dissent and z_R is the right dissent, then we need two conditions to be satisfied: (1) $F(m) - F(m-z_L) = F(m+z_R) - F(m)$ and (2) $\max\{F(m-z_L), 1-F(m+z_R)\} < F(m) - F(m-z_L)$. One can show that these two conditions together imply that $F(m-z_L) < 1/4$ and $F(m+z_R) > 3/4$.

[19] We have so far used WM beliefs at a critical dissent level $z > 0$. In principle, if we let $z = 0$, these beliefs turn to depend solely on exogenous signals. With such beliefs, one can show that for appropriate costs and signal strengths, there exists a two-party equilibrium where a high quality candidate enters at m with probability 1 while its low quality counterpart randomizes between m and OUT (see Section 4.3 for more on this with $n = 2$). However, this equilibrium is also very fragile in its requirements to block future entries on the flanks when one considers general distributions of voter's ideal policies (see Remark 2).

situation. Since he is randomizing between these two platforms, it follows that $\mu = \frac{1+2c}{2+2c}$. Hence $\mu < 1$ for all values of c. Given the MLRP property of signals, $\mu < 1$ can hold if and only if there is a cut-off value of the signal $\bar{s}_k \in \mathcal{S}$ such that candidate 2 at $m - z$ wins if and only if he generates a signal higher than this cut-off. This immediately implies that the constituency strictly prefers a voter-pandering party prior to these exogenous signals.

4 Two-candidate Contests

Section 3 established conditions under which elections select exactly two contesting parties (out of infinitely many candidates) and transmit information about the qualities of each contestant over and above what exogenous signals can provide. We now consider what happens when the number of candidates is 2. Following Duverger's Law, this is not only the most analyzed case in the literature but the median voter remains decisive for all distribution of voters ideal points and all sets of competing policies. Hence in this section, decisive constituency and the median voter are synonymous.

In line with the equilibrium characterized in Proposition 1, we assume WM beliefs for some $z > 0$ and focus attention on a strategy profile where high quality candidates enter with probability 1 at any vacant platform in the set $\{m - z, m + z\}$ while their low quality counterparts randomize between a vacant platform in this set and staying OUT unless candidate 2 observes a no-entry outcome in period 1 where then candidate 2 takes any platform to be declared an uncontested winner. We have the following result.

Proposition 2. *Suppose $n = 2$. There exists an equilibrium where* either *candidate 1 is an uncontested winner with a platform from the set $\{m - z, m + z\}$* or *candidate 2 is an uncontested winner with any platform* or *there is a two party contest, and all three outcomes are obtained with positive probabilities. While a two party contest can yield policy convergence,* (a) *each party stands on a platform from the set $\{m - z, m + z\}$,* (b) *it is partially revealing and* (c) *the pre-signal expected quality of contestant 1 is strictly less than that of contestant 2.*

While the equilibrium we characterize in Proposition 2 and that characterized in Proposition 1 share a common feature that in any two-party contest each contesting party chooses a platform that is a distance of $z > 0$ away from the ideal policy of the median voter, there are a number of important distinctions. With unbounded candidates, a two party contest is obtained with probability 1 while this probability is strictly less than 1 with two candidates. Second, with two candidates one can obtain outcomes with positive probability where both parties stand at a common but dissenting platform, while any two party contest with unbounded candidates yield policy divergence. However the most interesting difference is information transmission. In the unbounded case once any two candidates have announced their platforms, the median voter can be indifferent between the two parties prior to receiving exogenous signals. Thus dissent across periods has identical information content. However with two candidates, we show that candidate 2 will be more likely to be of high quality so that prior to receiving exogenous signals, the median voter must strictly prefer him to candidate 1. The intuition is as follows. Note that for this to be an equilibrium, the low quality candidates must be indifferent to staying out and entering (with dissent). However, the low type candidate in period 1 faces a positive probability on the equilibrium path of play that in period 2, the candidate is a low type and stays out in which case she wins unopposed. But on the equilibrium path of play with history of candidate 1 contesting on platform $m + z$, the only case when candidate 2 randomizes, he faces zero probability of winning uncontested. Hence, if the median voter would weakly prefer candidate 1 at $m + z$ to candidate 2 at $m - z$ then the expected equilibrium payoff of 1 had to be strictly larger than that of candidate 2. However that cannot be since both are randomizing between entry and OUT that yields a zero payoff. Hence it must be that once the two candidates stand at their respective dissenting platforms, it is more likely that candidate 2 wins. This is possible only if exogenous signals are expected to be more favorable for candidate 2, a situation that can happen only if the interim belief of the median voter is favoring candidate 2. It follows therefore that the probability of low quality participation in period 1 is higher than in period 2.

Our analysis in Section 3 implies that one should put more attention on equilibria where voter pandering is not obtained, particularly if we are concerned with future entries with general distributions of voters ideal point. To this end we have focussed in Proposition 2 on the equilibrium where on the equilibrium path of play there is a strictly positive probability that the platform m is not obtained (in fact this probability can be made equal to 1 by assuming that if uncontested, candidate 2 enters the competition and randomizes over the entire set $\mathbb{R}$). However, one can show that under generic conditions (not too different from those for Proposition 2) an equilibrium exists where in each period a low quality candidate randomizes between $m + z$ (or $m - z$) and m while a high quality candidate continues to announce platforms in the set $\{m - z, m + z\}$ with probability 1. Interestingly such an equilibrium can exist when prior to receiving exogenous signals, the median voter is indifferent between two dissenting parties.

4.1 Finitely many candidates and limit equilibrium

The equilibrium reported in Proposition 2 does not change qualitatively if there are more than two but a finite number of candidates. Unless candidate n faces a history of no entries where then he enters at any platform, all entry will be in the set $\{m - z, m + z\}$ and only the low quality candidates will stay OUT with some probability. One important difference though is that there will be no policy convergence due to the threat of future entry unless of course the second entry in the set $\{m - z, m + z\}$ is by the last candidate. In that sense, it is more likely that the equilibrium policies will look more like those in Proposition 1. Also, conditions for blocking future entry once two parties have entered the contest will be similar to those in Proposition 1.

With $n > 2$ one obtains a richer characterization of the temporal dimension of the voter's preferences over contestants. Recall that when $n = 2$, existence of equilibrium requires that the median voter strictly prefers party n $(= 2)$ to party 1 prior to receiving exogenous signals. With $n \geq 3$, this temporal pattern of the median voter's pre-signal preference becomes more involved. To see this, pick any candidate $1 \leq i < n - 1$ and assume i is facing an empty history. If he is of type L, then he is randomizing between an element in $\{m - z, m + z\}$ and OUT. For each such i, the expected payoff from standing in the set $\{m - z, m + z\}$ must equal zero that is the payoff obtained by staying OUT. However, as all players are following the same strategy, each i faces with positive probability the event that once they stand in the set $\{m - z, m + z\}$, they remain uncontested. If we denote this probability by $\mu(i)$, it is easy to check that $\mu(i) < \mu(i + 1)$. Since in each such event candidate i wins uncontested, their expected payoffs from standing in the set $\{m - z, m + z\}$ can remain all zero only if conditional on the event that i has a competitor (that occurs with probability $1 - \mu(i)$), i is more likely to win than $i + 1$. This can happen only if the constituency strictly prefers i to $i + 1$ prior to receiving exogenous signals. Now consider $i = n - 1, n$ when they face an empty history. This is a sub-game that is identical with the full game analyzed in Proposition 2. Hence as in that proposition it must be that the constituency strictly prefers n to $n - 1$ prior to receiving exogenous signals. It is easy to finally conclude that since for each candidate $i < n$ we have $\mu(i) > 0$, prior to receiving exogenous signals, it follows that as a first entrant the voters strictly prefer i to $i + 1$ for each $i = 1, \ldots, n - 2$. However, once some candidate $i < n$ has entered, if the second entrant is $j > i$ then the voters strictly prefer j to i. Moreover for any pair of two-party equilibria (i, j) and (i, j') where the two candidates are i and j and i and j', with $i < n$, the voters are indifferent between (i, j) and (i, j').

From this it follows that a two-party contest where candidate 1 enters with a platform $m + z$ is most liked by the MV and the expected quality of the 'first' entrant is higher the earlier such an entry takes place. However as in Proposition 2, the expected quality of the second entrant is always higher than the first. Also, as the number of candidates n rises, the expected quality of early first entrant rises while that of the second entrant remains fixed. Put together, it shows that in the limit when n approaches infinity, the MV becomes indifferent between any two dissenting entrant as established in Proposition 1 for the case of n being unbounded. We summarize these findings in the following corollary.

Corollary 2. *For any $n > 2$ and finite an equilibrium exists with at most two contestants and a two-party outcome is obtained with positive probability. In any outcome in the support of the equilibrium strategy profile, unless candidate n remains uncontested (in which case any policy can be obtained) the parties contest with platforms $m - z$ and $m + z$ for some $z > 0$. While a high quality contestant does so with probability 1 its low quality opponent randomizes between entering and staying OUT. The median voter strictly prefers early first entry but once such an entry takes place she is indifferent between the timing of the second entrant; however she strictly prefers the second entrant to the first. Finally, as $n \to \infty$, this equilibrium converges to the equilibrium with $n = \infty$ as reported in Proposition 1.*

Corollary 2 and Proposition 2 lead to the following remark concerning some novel possibilities in the literature that are open to empirical investigation.

Remark 3 (Period-specific beliefs and temporal dimension to extremism)**.** *We have assumed that the cutoff dissent in the definition of the WM beliefs was fixed over time. From a theoretical viewpoint this can be varied across candidates (or periods). One possibility that Corollary 2 and Proposition 2 allow us to do is to meet the temporal and history dependent preference of the median voter in the proposed equilibrium by varying the dissent cutoffs, without changing the probability of participation of low quality candidates. For example, we have shown that early first entry is strictly better. This translates to the possibility that later the first entry occurs more extremism is observed from such a first entrant. It also suggests that for any given first entrant, the second entrant is always less extremist than the first. In this sense, while a challenger is always less extreme, late initiation of political contests can lead to higher overall extremism.*

4.2 Limits on prior

The equilibria where two parties enter with low quality parties randomizing between entry and staying OUT lead to a sequence of interim beliefs that may or may not remain stable over periods and histories (Proposition 1 suggests these beliefs can be stable while Proposition 2 shows they must vary). Given these beliefs, how do equilibrium outcomes change as γ approaches 0? Note that to sustain these equilibria we need these beliefs to remain as they are in these propositions when γ is allowed to fall. But this is possible only if participation from low quality candidates falls as well. This implies that when γ is arbitrarily small, there will still exist a two-party equilibrium where almost certainly all contesting parties will be of high quality and at the same time since the interim belief is fixed it will yield political polarization with probability that approaches 1 when n approaches ∞. This suggests that even if the Osborne conjecture turns out to be correct (as it indeed is for $n = 2, 3, 4$), there can be a large discontinuity with respect to incomplete information since γ arbitrarily small is akin to almost full information. Of course over time, entry will be expected to be highly separated in time because the proportion of high quality candidates is small and the participation rate of low quality candidates is small as well.

4.3 Entry as a signaling device

We have so far used policy-driven WM beliefs to show that ideological dissent can signal strength and be sustained as equilibrium outcomes. We next ask if the decision to enter the political arena alone, irrespective of ideological positioning, can signal high quality? When one drops WM beliefs and instead makes the constituency care only about public signals, there arises a strong tendency to move towards the constituency's ideal point m for reasons well known in the HD paradigm. Hence the relevant question is about whether any form of signaling can take place without dissent? Interestingly the answer is yes and we make this point by studying the two-candidate game (i.e., $n = 2$).

Proposition 3. *Suppose $n = 2$. A partially revealing equilibrium exists where all type H candidates enter at m with probability 1 while all type L candidates randomize between*

m and OUT unless candidate 2 finds an empty history when she enters anywhere. As in Proposition 2, in any two party outcome, the median voter strictly prefers candidate 2 to candidate 1.

Proposition 3 highlights the fact that dissent is not necessary in general to signal quality but what is key is the fact that the risk of bearing the cost c of not being in the winning set can push low quality contestants out of the political market with positive probability when post-policy signals are strong enough. This kind of equilibrium behavior can be sustained through at least two types of beliefs off-the-equilibrium paths of play. For example, suppose voters believe that any outcome that is not m implies the deviating candidate is of type L with probability 1. Then it is clear that no deviation can be profitable. But such beliefs may be unreasonable as they suggest that apart from the median platform, voters can commit not to consider exogenous signals. Hence we prove Proposition 3 using beliefs that are on the other extreme where voters only use exogenous signals as it makes the equilibrium more compelling. As mentioned in footnote 19 in Section 1 however, Proposition 3 concerns only two candidates as potential competitors and the result may not be readily generalizable to an arbitrary n. This is because the conditions required to block future entry (on the flanks) become too restrictive if there are sufficiently many high quality candidates in the population. And most importantly (c/f. Remark 2), if we allow for a general distribution of voter ideologies, then any current outcome where two entrants stand at a common platform becomes extremely fragile as nearly all voters (located on the two sides) prefer other platforms so that future entries close on both sides of m can quickly capture the support of these original candidates at m.

It is important to note here that while any two-party outcome in Proposition 3 is ex-post pooling, the equilibrium is indeed partially separating as highlighted by the fact that a one-party outcome can be obtained if and only if candidate 1 stays OUT, thereby revealing himself to be of low quality. [20]

4.4 When extremism attracts signals

The model studied so far assumes that irrespective of the ideological promises, exogenous signals about the quality of each contestant arrive with certainty. But as mentioned earlier, there is substantial evidence that media coverage of moderate parties is less than their extremist counterparts. For example, McCluskey and Kim (2012) examined the coverage of 208 political action groups in 118 newspapers in the United States to conclude that "groups that expressed more polarized opinions on political issues were mentioned in larger newspapers, appeared earlier in articles, and were mentioned in more paragraphs". We show that when one incorporates this finding into our model assumptions with the view that public signals are after all disseminated through press coverage of the electoral process, informative equilibria with ideological dissent become both more intuitive and realistic.

[20] In the appendix we also prove that pooling equilibria can exist in principle but again they will be too fragile in a model of free entry.

To do so, we modify the benchmark model in the following way. Let $Q(z)$ be the probability with which exogenous signals are generated by an entrant with position $x \in \{m-z, m+z\}$. We assume that $Q(\cdot)$ is strictly increasing in the interval $[0, z^*]$ for some $z^* > 0$ with $Q(0) = q$, $0 \leq q < 1$ and $Q(z) = 1$ for all $z \geq z^*$.

The benchmark model we have studied so far assumes $Q(z) = 1$ for all $z \geq 0$. In that case we have shown (with $n = 2$) that a partially revealing equilibrium exists (viz. Proposition 3) where a high quality candidate enters with probability 1 at m, a low quality entrant does so only with some probability and otherwise stays OUT of competition, and voters simply use signals to update posteriors. In the following proposition we show that such equilibria cease to exist when signal probabilities are endogenously linked with the announced platform while a partially revealing equilibrium with dissent can still be obtained with this modification. While the proof of the proposition uses the assumption that $q = 0$ the result continues to holds even if $q > 0$ but small.

Proposition 4. *Let $n = 2$ and suppose exogenous signals are strong but arrive with probability $Q(z)$ as defined above. There exists $z^* > 0$ such that for all $0 < c < 1$ if signal precision is strong enough and if voters only look at signals to form beliefs, then in any equilibrium a high quality candidate must show dissent with positive probability.*

The intuition of the above result is straightforward. Suppose voters' beliefs are not based on platform choices but depend solely on the information they obtain from exogenous signals (as assumed in Proposition 3). If both entrants stand at m then the probability that the signal technology will operate is small (zero in the proof) so that each candidate obtains a payoff close to (equal to in the proof) 1/2. From this point, consider a high quality candidate deviating to the minimal dissent level z^* where the probability that it generates a signal for itself is 1. For this deviation to be profitable, the voters must strictly prefer him at z^* than his rival at m who does not reveal any additional signal and therefore believed to be of type H with probability γ, the common prior. For this, he must generate a signal that is strong enough to cover the risks of losing and incurring the cost c. We show that for any value of this cost, profitability of this deviation is upheld if the informative power of exogenous signals is relatively strong. Hence we accept that the equilibrium reported in Proposition 3 is fragile in face of the possibility that extremism attracts signals.

Remark 4. *It is important to note that in this Q - framework, one can obtain equilibria where strategic dissent appears and signals strength: high quality entrants can find an appropriate dissent $z' > 0$ by balancing the electoral cost of ideological dissent with the informational advantage of revealing favorable signals, while low quality counterparts randomize between entering with dissent z' and staying OUT or standing for the voter pandering policy m. These observations put together establish the central position of our paper: dissent is endogenous, strategic, and a natural feature of political competition when candidates have hidden qualities even when they are otherwise purely office-seeking politicians.*

While a more complete analysis of the model with signal probability function Q is beyond the scope of this paper, a promising line of enquiry is to consider a framework where the press spends resources to emit signals about candidate quality. One can then ask whether a press driven either by an ideological bias or by size of viewership would actually spend more resources behind candidates who announce unpopular policies. We reserve this line of research for the future.

5 Strategic timing

We have assumed that there is an exogenous process that ascertains the order in which candidates line up in the entry game. While the sequential entry model, albeit exogenously fixed in our case, is indeed a better representation of entry games in real politics than what models of simultaneous entry offer, it is not enough to address the choice by parties about when to announce their candidature. Surprisingly, the literature on candidate positioning and entry is largely silent about this very important aspect of elections. We therefore investigate the impact of endogenizing the order of entry on the quality of contestants and degree of ideological dissent. In doing so we ask if this makes voters more informed about their political alternatives or if they are better (worse) off through increased policy moderation (extremism).

In order to address this, we make the following modifications to the model. We assume that there are a finite number $n \geq 2$ of potential entrants, but there is no sequence in which they have to decide whether to enter or not. Instead each candidate can choose to enter in any 'period' $t = 1, 2, \ldots$ with a potentially infinite horizon. Keeping in line with the idea of WM beliefs, we will assume that in each period there is a cutoff dissent $z_t > 0$ such that the voters would believe an entrant in period t to be of type L with probability 1 whenever the dissent shown by the entrant is less than z_t. We have the following result.

Proposition 5. *Suppose a finite number $n \geq 2$ of candidates (each of whom is of type H with probability γ) are free to enter in any period, wait to announce entry for the next period, or quit the contest. Then there exists an equilibrium where early entry signals high quality but must come with higher ideological extremism. The equilibrium dynamics have the following features:*

1. *If γ is high then (a) there is a sequence (of length ≥ 1) of early entries where only high quality candidates enter by randomizing between entering with dissent $z > 0$ and waiting, followed by (b) either (b.1) a sudden announcement of quits from all waiting candidates when the number of currently contesting candidates exceeds some threshold n^* (where $n^* = 2$ when c is high),* or *(b.2) the number of currently contesting candidates is below n^* and the game has at most a two period terminal sequence of late entries where in the first period all currently waiting high quality candidates (if any) enter with probability 1 and dissent $z' < z$ while all low quality candidates randomize between entering with dissent z' and quitting the contest; if and only if there is still no contestant at the end of the first period in this terminal sequence (which is possible only with probability $(1-\gamma)^n$), all candidates reveal themselves as type L and enter in the final period with probability 1 at m;*

2. *If γ is low then all entries take place in at most two periods: In period one all high quality candidates enter with dissent z while all low quality candidates randomize between entry with dissent z and waiting. If there are n^{**} contestants at the beginning of period 2, then a fraction $\phi(n^{**})$ of the remaining $n - n^{**}$ candidates enter at m while others quit, where $\phi(n^{**})$ is decreasing in c and signal strengths so that if c and signal strengths are high enough then $\phi(n^{**}) = 0$ for each $n^{**} \geq 2$.*

If we focus on a 2-party contest, we first note that the equilibrium reported in Proposition 5 allows for that provided costs and signal strengths are appropriate. The contests can be between (i) two type H candidates who reveal themselves through early entry and equally unpopular platforms, or (ii) two type L candidates who reveal themselves by very late entry and a common voter pandering platform or (iii) a candidate with known quality (an extremist H who enters early or a populist L who enters very late) and a candidate with intermediate degree of extremism but whose identity is unclear. However the equilibrium does not permit a 2-party contest between a known type H extremist and a known type L populist.

It is important to note here that even if there are at least 2 type H candidates in the population and the costs and signals are such that future entry is blocked whenever at least two entries take place in the early periods, the game can still continue for a long time. This is because in these early stages the type H candidates randomize between entry and waiting so that in each such period there is a strictly positive probability of obtaining no entries. In addition, the equilibrium in general involves a number of other novel features and the proof (moved to the appendix) has many subtle steps and implicit assumptions that we now endeavor to make explicit.[21]

First, we do not explicitly incorporate time discounting and hence in principle there is no reason for the equilibrium proposed to be implemented in any finite time. But this is a minor issue for the larger scope of this result and we impose an ad-hoc no-delay criterion so that whatever is described in the strategy profile gets implemented from period 1. As entry timings are now endogenously determined, there is a certain degree of simultaneity in decisions in each period and the length of the game itself is an equilibrium feature. In this regard, the WM beliefs and the common ideal policy m of the constituency play an important role by allowing us to avoid the well-known Nash equilibrium existence problems in spatial games with multiple players.

Given this, the basic idea behind the construction of the equilibrium stems from the following observations. First, when the prior belief γ is very high, it is best for the low-quality candidates to wait and learn from political activities during the early phase of the entry game. Hence early entry signals quality with probability 1 and when voters choose

[21] While we are by no means reporting a full analysis of the equilibrium set with endogenous timing and instead focussing on equilibria with dissent, we note that there can be another equilibrium where players only use time to signal quality. In that equilibrium, a given number of high quality candidates stand in period 1 at the voter pandering platform m, voters do not use WM beliefs and break the tie by endorsing the candidate with the highest signal and all other candidates immediately QUIT. The number of contestants is determined by the cost c.

between a set of ‘early entrants’ who are all ‘known’ to be of high quality they must use a tie-breaking rule. Here we make voters pick that candidate (and vote for him if there is no other better candidate with possibly lower dissent and relatively strong signal) who produces the highest signal. This tie-breaking rule is crucial for the existence of our equilibrium as it thwarts entry from low-quality candidates with probability 1 during the early stages if costs are high enough. However as γ is high and the surplus in the political market is bounded, even high quality candidates play it safe and randomize between entering and waiting. WM beliefs in addition ensure that whenever they enter they do so with the cutoff dissent $z > 0$.

This brings the process to its next period. If too many high quality candidates end up entering in period 1 then there may not be enough room for any further entry and the game stops. Otherwise such entry (including no entry as a realization of the randomization in period 1) necessarily reduces the current-period prior about waiting candidates to be of high quality. If this prior is still high enough then existing high quality candidates continue to find it in their best interest to randomize between entering with dissent z as their predecessors did and wait while low quality candidates continue to wait. This process unfurls itself and at any of these successive periods the game can stop with positive probability when there are too many entrants in the history, all ‘known’ to be of high type. Otherwise a period comes when the current prior about the waiting candidates falls below some cutoff (that is history dependent) at which high quality candidates gain enough confidence to enter with certainty. At this stage the game can last for at most two rounds. With a low current prior, low quality candidates are able to randomize between mimicking their current high quality counterparts and waiting. And this mimicking ensures that the pre-signal posterior of these late current entrants are strictly less than 1. If history already has some existing contestants who have revealed themselves as high quality and with dissent z, then these current entrants, even if of high quality, cannot win against these high quality predecessors unless their ideological dissent is smaller. Thus candidates who enter at this stage with an existing set of past entrants (who are all known to be of high quality) do so with dissent $0 < z' < z$ so that their platform commitments exhibit ideological moderation.

Given play continues on the equilibrium path, the final stage appears when all currently waiting candidates end up revealing themselves as low quality by the act of waiting for ‘too long’. Hence they either quit the contest at this stage or contest with the populist platform m. Note here that for them to contest with platform m against a rival who is at $m + z$ (and therefore has revealed himself to be high quality), it must be that $z \geq h$. Since $z > h$ can never be obtained in equilibrium, we assume without any harm that $z < h$ so that the equilibrium remains parametrically robust. This implies that whenever there is an early entry with dissent z, there will never be a stage where some candidate contests with a populist platform. This is of course not true if the only contestants in the history commits to platforms with dissent z' as the pre-signal beliefs for such contestants are bounded away from 1.

In summary, while dissent keeps signaling quality, the feasible range of dissent in order to achieve this gets moderated over time. This is an aspect of equilibrium behavior that is not a necessity when timing is exogenous.

6 Conclusion

We analyzed a Downsian model of sequential entry and incomplete information about candidate quality both when entry timings are exogenously given and endogenously determined. We conclude by first summarizing all our results and then discussing some generalizations of the framework in which these results remain robust.

The central message of our analysis is as follows: ideological dissent can be an equilibrium feature with good quality candidates using dissent to signal their type while low quality candidates randomize between competing with dissent (to hide their types) and staying out. In particular, we have characterized conditions for the dissent equilibria under exogenous entry in which two-party contest is obtained as suggested by Duverger's Law. Interestingly, the presence of partially informative signals from exogenous sources can keep low quality candidates out of competition to a certain degree and electoral competition helps voters learn about candidate types beyond what these signals can reveal. However, the very presence of these signals fosters an endogenous mechanism of strategic information transmission that has a polarizing impact on political representation. As signals get more precise, it reduces this polarization but surprisingly increases participation from low quality parties. Further, the timing of entry determines extremism to an extent. When the order of entry is determined endogenously, there is an equilibrium in which high quality parties typically enter early in the entry game with higher degree of dissent (than other high quality parties who enter late) so that over time if entry continues to take place, there is abrupt policy moderation. While other papers have derived extremism in policies, we do so in a model of endogenous political entry. Thus, we combine a model of political entry (which typically pays little attention to agency issues) with a signaling model (where usually the political competition is modeled in a very reduced form).

We have used what we called Weak Monotone beliefs which assumes that there is a unique pair of policies symmetrically around the median point such that voters believe candidates taking ideological stances that are less extreme than these points to be high types with probability zero but at or beyond these points assigns some positive probability of being a high type that does not increase with the degree of dissent. In other words, these beliefs are step functions. All the qualitative features of our results will continue to hold if we allowed beliefs to be continuous in the degree of dissent. Smooth beliefs would entail a high quality entrant to find an 'optimal' dissent that balances between the risk of a bad exogenous signal versus the gain in pre-signal voter confidence.

It is worth noting that while we have laid down conditions for existence of two-party contests (excepting for the case with endogenous entry where we are more general), it is straightforward to see that if costs are lowered, one can make room for more entry. However the qualitative feature of strategic dissent will continue to hold and the number of entrants will remain small due to entry costs and the presence of informative exogenous signals that limit the success rate of low quality entry.

We have assumed throughout that the only way to signal quality is via policy choice. In reality, campaign expenditure can also do the job. In that case, high quality candidates would want to spend more as with more spending they may be able to get their message across more clearly or even get more precise press endorsements. And given this is a costly signal, the fear of an informative press may keep the low quality candidates from matching the expenditures of their high quality counterparts. In reality, there can be limits on campaign spending by law, possibly because such spending may be socially suboptimal in which case policy dimension is the main avenue of signaling. It may also be the case that new entrants can raise less money than incumbents in which case they may try to make up for it by taking extreme positions (if they are high quality) compared to the incumbent to generate more precise signals. At the other extreme, if candidates have unlimited resources to fund campaigns (so that they do not care about expected losses from campaign expenses), ideological extremism stands out as a natural avenue to signal strength.

7 Appendix

Weakly Monotone beliefs: Consider an arbitrary belief function $\rho_k : \mathbb{R} \to [0,1]$ that assigns a probability $\rho_k(x_k)$ to the event that $\theta_k = H$ when contestant k is seen to have committed to platform $x_k \in \mathbb{R}$. We say that ρ_k satisfies Weak Monotonicity (WM) if there exist $0 < \rho \leq 1$ and $z_k > 0$ such that

$$\rho_k(x_k) = \begin{cases} 0 & \text{if } x_k \in (m - z_k, m + z_k) \\ \rho & \text{otherwise} \end{cases} \tag{2}$$

To sustain WM-beliefs in equilibrium, one requires the size ρ to be Bayes-consistent with equilibrium strategies and well-defined on out-of-equilibrium paths of play. To fix ideas, pick $z_k > 0$ and a strategy profile σ^n where, if history permits, and if candidate k in question is of type H then it enters the contest by committing to a platform from the set $\{m - z_k, m + z_k\}$ with probability 1. On the other hand, if the candidate is of type L, then it stays OUT (or enters the contest at m) with probability $1 - r_k$ and enters the contest in the set $\{m - z_i, m + z_i\}$ with probability r_k. An out-of-equilibrium play if, all 'other' players follow σ^n, would be for candidate k to announce platform commitment $y \notin \{m - z_i, m, m + z_i\}$ (or $y \notin \{m - z_i, m + z_i\}$) depending on the equilibrium concerned). Bayes rule and the WM property of beliefs would then imply the following: if players (candidates and voters) observe an ideology announcement x_k from contestant k, then their belief is

$$\rho_k(\sigma^n | x_k) = \begin{cases} 0 & \text{if } x_k \in (m - z_k, m + z_k) \\ \frac{\gamma}{\gamma + r_k(1-\gamma)} & \text{otherwise} \end{cases} \tag{3}$$

The role of the exogenous signals is to refine the voters' beliefs further. Suppose contestant k reveals a signal $s_k \in \mathcal{S} \subset \mathbb{R}$. Then,

$$\rho_k(\sigma^n | s_k; x_k) = \begin{cases} 0 & \text{if } x_k \in (m - z_k, m + z_k) \\ \frac{\phi(s_k|H)\rho_k(\sigma^n|x_k)}{\phi(s_k|H)\rho_k(\sigma^n|x_k) + \phi(s_k|L)(1-\rho_k(\sigma^n|x_k))} & \text{if } x_k \notin (m - z_k, m + z_k) \end{cases} \tag{4}$$

Proof of Proposition 1:

Consider the following strategy profile denoted by σ^∞ defined for some $z > 0$:

- At any period with a history with no existing contestant: enter at $m+z$ with probability 1 if of type H and otherwise enter at $m+z$ with probability p and stay OUT with probability $1-p$;
- At any period with history where platform $m+z$ is taken but $m-z$ is vacant: enter at $m-z$ with probability 1 if of type H and otherwise enter at $m-z$ with probability p and stay OUT with probability $1-p$;
- At any period with history where both $m+z$ and $m-z$ are taken: stay OUT;
- For any other history in any period: play as if it is a history with no existing contestant.

Let $\Pi(\theta_i, \theta_j|\sigma^\infty)$ be the expected payoff of candidate i of type θ_i when i contests with platform $x_i = m+z$, j contests with platform $m-z$ and is of type θ_j when all players follow σ^∞. Note that along the equilibrium path of play, all candidates and voters have common belief about contestants, that is,

$$\rho_j(\sigma^\infty) = \rho = \frac{\gamma}{\gamma + p(1-\gamma)}.$$

For any conditional probability function ϕ that satisfies MLRP, it must be that $\Pi(L, H|\sigma^\infty) < \Pi(L, L|\sigma^\infty)$. Note that $\Pi(L, L|\sigma^\infty) = \frac{1-c}{2} > 0$ if the voter is indifferent between two contestants at $m-z$ and $m+z$. Given z is fixed with the WM beliefs, this means the randomization probability is fixed as well. While in general one can obtain equilibria where randomization probabilities vary over period, we will focus on stationary probabilities. Therefore, as i is randomizing in accordance with σ^∞ and his payoff from staying OUT equals 0, it must be that

$$\rho\Pi(L, H|\sigma^\infty) + (1-\rho)\left(\frac{1-c}{2}\right) = 0. \tag{5}$$

Also since $0 < \gamma < 1$ and we want $0 < p < 1$, it follows that $0 < \rho < 1$. That holds if and only if $\Pi(L, H|\sigma^\infty) < 0$.

Next we need to block deviations which are of two types: (a) repositioning or exit of contesting players and (b) other entries. We begin with deviations from contesting players. It is straightforward to confirm that (5) implies that on the equilibrium path of play, a high quality contestant earns strictly positive expected payoff and hence will not exit. Similarly, a low quality contestant earns 0 on the equilibrium path and has no incentive to move probabilities towards the action OUT. Also by virtue of WM beliefs defined for the cutoff dissent z, the only deviation that requires attention is where a type L contestant deviates to m. So suppose there is such a deviation and with no loss of generality suppose this is by player i. Assume first that following this deviation, the rest of the players continue with σ^∞. This means that in the continuation sub-game there will be exactly two future entrants, say j and k with $x_j = m + z$ and $x_k = m - z$. Also note that given WM beliefs, player i is believed to be of type L with probability 1 by the voters. Let $\Pi_L(m, \theta_j, \theta_k|\sigma^\infty)$ be the expected payoff of candidate i when j contests with platform $x_j = m + z$ and is of type θ_j and k contests with platform $m - z$ and is of type θ_k given all players follow σ^∞. Note that

$$\Pi_L(m, H, H|\sigma^\infty) < \Pi_L(m, L, H|\sigma^\infty) = \Pi_L(m, H, L|\sigma^\infty) < \Pi_L(m, L, L|\sigma^\infty).$$

Note that in this continuation path, the beliefs continue to be ρ for contestants j and k. Hence to block this deviation the expected payoff of player i from this deviation must be non-positive, that is

$$\rho^2\Pi_L(m, H, H|\sigma^\infty) + 2\rho(1-\rho)\Pi_L(m, L, H|\sigma^\infty) + (1-\rho)^2\Pi_L(m, L, L|\sigma^\infty) \leq 0. \qquad (6)$$

To study sequential rationality in the continuation game with history $\{m\}$, we first pin down conditions under which there is no further entry in the continuation game with history $\{m - z, m + z\}$. So consider the history $\{m - z, m + z\}$. To block entry in the interval $(m - z, m + z)$, notice that the most profitable entry here is at m. If there is no further entry after the third entry at m we are considering now, then the conditions to block this entry at m is what is given under (6). Next observe that if this entry at m is blocked when there is no further entry in the future, then it will also be blocked with further entry as future entries cannot help the prospects of this entrant at m. So consider entry on the flanks, that is outside the interval $(m - z, m + z)$. The most profitable entry in this case in the set $\{m - z, m + z\}$ and by a candidate with type H. as before, let $\Pi_H(m + z, \theta_j, \theta_k|\sigma^\infty)$ be the expected payoff of candidate i of type H standing at $m + z$ when j contests with platform $x_j = m + z$ and is of type θ_j and k contests with platform $m - z$ and is of type θ_k given all players follow σ^∞. Notice like before we have

$$\Pi_H(m + z, H, H|\sigma^\infty) < \Pi_H(m + z, L, H|\sigma^\infty) = \Pi_H(m + z, H, L|\sigma^\infty) < \Pi_H(m + z, L, L|\sigma^\infty),$$

and blocking this entry requires

$$\rho^2\Pi_H(m+z, H, H|\sigma^\infty)+2\rho(1-\rho)\Pi_H(m+z, L, H|\sigma^\infty)+(1-\rho)^2\Pi_H(m+z, L, L|\sigma^\infty) \leq 0. \quad (7)$$

Notice that given the WM beliefs, as (7) blocks entry at $m + z$, it will also block further entry as future entries can only hurt the prospects of current entrants.

We are now in a position to address sequential rationality in the continuation subgame with history $\{m\}$. Given conditions (6) and (7) and the definition of the strategy profile σ^∞, the continuation game with history $\{m\}$ yields an eventual outcome where there are exactly three contestants taking positions $m - z$, m and $m + z$ such that: the contestants at $m - z$ and $m + z$ are of type H with probability ρ (prior to signals). Let $\Pi_L(m + z, \theta_j|\sigma^\infty)$ be the expected payoff of candidate i of type L standing at $m + z$ when j contests with platform $x_j = m + z$ and is of type θ_j and k contests with platform m and is therefore of type L given all players barring k follow σ^∞. Since player i is randomizing between $m + z$ and OUT, we need

$$\rho\Pi_L(m + z, H|\sigma^\infty) + (1-\rho)\Pi_L(m + z, L|\sigma^\infty) = 0. \qquad (8)$$

Conditions (5) and (8) suggest a knife-edge property of the equilibrium under construction since, given z and p are fixed over time, ρ remains stationary as well. This means existence of this equilibrium requires that

$$\frac{\Pi(L, L|\sigma^{\infty})}{\Pi(L, H|\sigma^{\infty})} = \frac{\Pi_L(m+z, L|\sigma^{\infty})}{\Pi_L(m+z, H|\sigma^{\infty})}. \tag{9}$$

While (9) can hold in general, the following claim shows that this condition is always satisfied if p, z, γ, h and σ^{∞} are such that prior to receiving exogenous signals, voters are indifferent between a contestant at $m+z$ (or $m-z$) and a contestant at m.

Claim 1. *Suppose prior to receiving exogenous signals, p, z, γ and h are such that voters are indifferent between a contestant at $m+z$ (or $m-z$) and a contestant at m, that it, $\rho_k(\sigma^{\infty}|x_k = m+z)h - z = 0$ Then condition (9) holds.*

To prove the claim it is enough to note that with the voter's pre-signal indifference, if an entrant of type L with platform $m+z$ defeats a type $\theta \in \{L, H\}$ entrant at $m-z$ then he also beats the player at m. Since ties on the other hand are zero probability events, the claim follows.

So what remains is that once a history $\{m-z, m, m+z\}$ is obtained, there is no further entry. But we have already blocked further entries from history $\{m-z, m+z\}$ so such an entry cannot become more profitable from the history $\{m-z, m, m+z\}$.

Hence for existence of this equilibrium, it is (a) sufficient that the voters have pre-signal indifference between a candidate at m and the contesting candidates (at $m-z$ and $m+z$), (b) necessary that $\Pi(L, H|\sigma^{\infty}) < 0$ from (5), (c) Condition (6) and (d) Condition (7).

We first look at Condition (6). It is easy to see that if the signal precision is strong, then given requirement (a) above, it is very unlikely that a candidate standing at m in off-the-equilibrium play can win against a high quality candidate with dissent. Hence the first two terms are indeed negative. On the other hand if the precision is at the same time not too strong, it is not very likely that a candidate standing at m in off-the-equilibrium play can win against two low quality candidates with dissent. Hence if the signal precision is intermediate, Condition (6) will hold. We will now show that intermediate precision on the other hand will mean that Condition (5) implies Condition (7). Given the voter's indifference as in point (a), let $P_{L>H}$ be the probability that a low quality entrant defeats a high quality entrant. By the MLRP of the signals, it follows that $P_{L>H} < 1/2$. Similarly define $P_{H>H}$ and $P_{L>L}$ and note that $P_{H>H} = P_{L>L} = 1/2$ by the voter's indifference.

Fact 1. *If $c > 1/2$ and $P_{L>H} > 1/3$ then Condition (5) implies Condition (7).*

To see why Fact 1 holds observe the following. Condition (b) above implies $P_{L>H} < \frac{c}{1+c}$. Next observe that (i) iff $P_{L>H} > 1/3$ then $\Pi_H(m+z, L, H|\sigma^{\infty}) < \Pi(L, H|\sigma^{\infty})$ and (ii) iff $P_{L>H} > 1 - 1/\sqrt{2}$ then $\Pi_H(m+z, L, L|\sigma^{\infty}) < \Pi(L, L|\sigma^{\infty}) = \frac{1-c}{2}$. Finally, if $c > 1/2$ then $1 - 1/\sqrt{2} < 1/3 < \frac{c}{1+c}$. We now show that if $\Pi_H(m+z, L, H|\sigma^{\infty}) < \Pi(L, H|\sigma^{\infty})$ and

$\Pi_H(m+z,L,L|\sigma^\infty) < \Pi(L,L|\sigma^\infty)$ then (5) implies (7). From (7) the above observations and the fact that $\Pi_H(m+z,H,H|\sigma^\infty) < \Pi_H(m+z,L,H|\sigma^\infty)$, it follows that

$$\rho^2\Pi_H(m+z,H,H|\sigma^\infty) + 2\rho(1-\rho)\Pi_H(m+z,L,H|\sigma^\infty) + (1-\rho)^2\Pi_H(m+z,L,L|\sigma^\infty) <$$

$$\rho(2-\rho)\Pi(L,H|\sigma^\infty) + (1-\rho)^2\Pi(L,L|\sigma^\infty) <$$

$$\rho\Pi(L,H|\sigma^\infty) + (1-\rho)\Pi(L,L|\sigma^\infty) + \rho(1-\rho)\Pi(L,H|\sigma^\infty) < 0$$

since from (5) we know that $\rho\Pi(L,H|\sigma^\infty) + (1-\rho)\Pi(L,L|\sigma^\infty) = 0$ and $\Pi(L,H|\sigma^\infty) < 0$. This establishes the Fact.

To end the proof of the proposition it is enough to observe that if c is close to one then no entry can be profitable. Thus we have shown that if $1/2 < c$ and $1/3 < P_{L>H} < 1/2$ then the equilibrium exists provided c is not too close to 1.

□

Example of a signal technology:

Given Proposition 1, we give an example of a signaling formulation to make the conditions for existence of the equilibrium therein clearer. Suppose then that for a contesting candidate i this signal s_i is a random variable taking values in the set $\mathcal{S} = \{\lambda\} \cup [0,1]$. The probability of obtaining $s_i \in \mathcal{S}$ is conditional on θ_i and is given by the respective conditional densities $\phi(s_i|H)$ and $\phi(s_i|L)$. These conditional densities have the following properties. Let $f(s_i|\theta_i)$ be such that for each $\theta_i \in \{H,L\}$ we have $\int_{s_i\in[0,1]} f(s_i|\theta_i)\mathrm{d}s_i = 1$. For some $0 < \epsilon < 1$ we assume (i) $\phi(s_i = \lambda|L) = 1-\epsilon$ and $\phi(s_i|L) = \epsilon f(s_i|L)$ otherwise and (ii) $\phi(s_i = \lambda|H) = 0$ and $\phi(s_i|H) = f(s_i|H)$ otherwise. Thus, if $\theta_i = L$ then the signal technology reveals this (through the $s_i = \lambda$) to the constituency with probability $1-\epsilon$, while with probability ϵ the candidate can be portrayed as one with good quality. However, high quality candidates are more likely to get more favorable signals so that $f(\cdot|\theta_i)$ satisfies Monotone Likelihood Ratio Property. The implications of this signal technology are as follows. When the signal received is $s_i = \lambda$, it implies a literal revelation of a candidate being of low quality. However, this is the only fully revealing information that the constituency can hope to obtain via exogenous signals. All other signals (from the set $[0,1]$) can at most provide partial information and a higher value of the signal in this set indicates stronger perception of high quality (the standard MLRP case). It is important to check that for any arbitrary pre-signal belief ρ the martingale property is satisfied, that is, we must have the expectation of the post-signal belief $\rho(s_i) =\rho$. Thus

$$\mathbb{E}[\rho(s_i)] = \rho(\lambda)(1-\epsilon)(1-\rho)+$$

$$\int_{s\in[0,1]} \frac{\rho f(s_i|H)}{\rho f(s_i|H) + (1-\rho)\epsilon f(s_i|L)}(\rho f(s_i|H) + (1-\rho)\epsilon f(s_i|L))\mathrm{d}s = \rho$$

as required since $\rho(\lambda) = 0$. Now if ϵ is small enough, then Condition (6) will hold as the third term in that condition will become small. On the other hand a small ϵ helps in the requirement of $P_{L>H} > 1/3$ in Fact 1.

Proof of Proposition 2:

We start with candidate 2. Fix $z > 0$ and define the WM beliefs with respect to z. Suppose candidate 1 is at $m+z$ and let ρ_1 be the pre-signal Bayesian belief (held by candidate 2 and the voters) that candidate 1 is of type H with $\rho_1 = \frac{\gamma}{\gamma+p_1(1-\gamma)}$ where $0 < p_1 < 1$ is the probability with which a type L candidate 1 stands at $m+z$ while with probability $1-p_1$ he stays OUT. Let $p_2(z)$ be the probability with which 2 plays $m-z$ and otherwise plays OUT. Let $\Pi(x_2, \theta_2|x_1, \theta_1)$ be the expected payoff of candidate 2 of type θ_2 when 1 is at x_1 and is of type θ_1. Then, indifference of 2 between $m-z$ and OUT when $x_1 = m+z$ is given by

$$\rho_1\Pi(m-z, L|m+z, H) + (1-\rho)\Pi_2(m-z, L|m+z, L) = 0. \tag{10}$$

Since $0 < p_1 < 1$ (so that $0 < \rho_1 < 1$ it must be that $\Pi(m-z, L|m+z, H) < 0 < \Pi(m-z, L|m+z, L)$. Also, a direct calculation yields

$$p_1 = -\frac{\gamma\Pi(m-z, L|m+z, H)}{(1-\gamma)\Pi(m-z, L|m+z, L)},$$

so that $p_1 < 1$ if and only if

$$\frac{1-\gamma}{\gamma} > -\frac{\Pi(m-z, L|m+z, H)}{\Pi(m-z, L|m+z, L)}. \tag{11}$$

To block a deviation by candidate 2 to a different platform, it suffices to ensure that on this path of play, candidate 2 does not benefit from deviating to the platform m and thereby revealing himself as of low quality. Thus we also require that

$$\rho_1\Pi(m|m+z, H) + (1-\rho)\Pi(m|m+z, H) \leq 0. \tag{12}$$

Now we look at candidate 1's entry decision and focus on low quality. By standing at $m+z$ his expected payoff is

$$\gamma\Pi(m+z, L|m-z, H) + (1-\gamma)[p_2(z)\Pi(m+z, L|m-z, L) + (1-p_2(m))]. \tag{13}$$

We need to block him from deviating to m. While this is off-the-equilibrium path, we need to specify the behavior of candidate 2 that will be sequentially rational in this sub-game. We assume that while a high quality player continues to stand at $m+z$ with probability 1, candidate 2 of type L randomizes between $m+z$ and m with probabilities $p_2(m)$ and $1-p_2(m)$. Then we need

$$\gamma\Pi(m|m+z, H) + (1-\gamma)\left(p_2(m)\Pi(m|m+z, L) + \frac{1-p_2(m)}{2}\right) \leq 0. \tag{14}$$

Finally we need to ensure that the behavior of candidate 2 at history m is sequentially rational. Since he is randomizing, it follows that we need

$$\Pi(m+z, L|m) = \frac{1}{2}. \tag{15}$$

Given that beliefs are monotonic in signals, (15) implies that this off-the-equilibrium randomization probability $p_2(m)$ must be sufficiently high so that prior to signals the voters prefer the candidate at m to $m+z$.

From (13) and the fact that $p_2(z) < 1$ we obtain

$$\frac{1-\gamma}{\gamma} < -\frac{\Pi(m+z, L|m-z, H)}{\Pi(m+z, L|m-z, L)}. \tag{16}$$

We first show that if prior to receiving exogenous signals the median voter is indifferent between candidate 1 at $m+z$ and candidate 2 at $m-z$, then this equilibrium cannot exist. To see this, look at conditions (10) and (13) and note that since prior to receiving exogenous signals the median voter is indifferent between a candidate 1 at $m+z$ and candidate 2 at $m-z$, it must be that $\Pi(m+z, L|m-z, H) = \Pi(m-z, L|m+z, H)$ and $\Pi(m+z, L|m-z, L) = \Pi(m-z, L|m+z, L)$. Hence, from (13) and the fact that $p_2(z) < 1$ it follows that condition (16) becomes

$$\frac{1-\gamma}{\gamma} < -\frac{\Pi(m-z, L|m+z, H)}{\Pi(m-z, L|m+z, L)}, \tag{17}$$

a contradiction with (11). So for existence we need in general that

$$-\frac{\Pi(m-z, L|m+z, H)}{\Pi(m-z, L|m+z, L)} < \frac{1-\gamma}{\gamma} < -\frac{\Pi(m+z, L|m-z, H)}{\Pi(m+z, L|m-z, L)}, \tag{18}$$

which holds if and only if

$$\Pi(m+z, L|m-z, H)\Pi(m-z, L|m+z, L) < \Pi(m-z, L|m+z, H)\Pi(m+z, L|m-z, L).$$

Notice that if the median voter strictly prefers candidate 2 at $m-z$ to candidate 1 at $m+z$ then $0 < \Pi(m+z, L|m-z, L) < \Pi(m-z, L|m+z, L)$ and $\Pi(m+z, L|m-z, H) < \Pi(m+z, L|m-z, H) < 0$ so that condition (18) holds. Finally suppose the median voter strictly prefers candidate 1 at $m+z$ to candidate 2 at $m-z$. Then $0 < \Pi(m-z, L|m+z, L) < \Pi(m+z, L|m-z, L)$ and $\Pi(m+z, L|m-z, H) < \Pi(m+z, L|m-z, H) < 0$ and condition (18) does not hold. This implies that since γ and z are fixed, it must be that $\rho_1 < \rho_2$ yielding $p_1 > p_2$. Barring this, it is easy to see that existence will require c to be high but strictly less than 1 and signal precision of intermediate level as in Proposition 1.

$\square$

Proof of Proposition 3:

Consider the following strategy profile:

- If of type H, enter at m with probability 1;
- If of type L, randomize:
  - if $i = 1$: enter at m with probability r_1 and stay OUT with probability $1 - r_1$;

– if $i = 2$: (a) facing an empty history, enter at any policy; (b) facing a history where m is occupied, enter at m with probability r_2 and stay OUT with probability $1-r_2$; (c) facing any other history, play a best response;

- Whenever possible, the constituency uses public signals to form Bayesian beliefs about quality of the entrants, irrespective of their ideological positions.

First observe that no candidate has an incentive to enter at any policy other than m given these beliefs. So it suffices to look at the equilibrium-path of play. Let $\Pi_2(L|\theta_1)$ be the expected payoff of candidate 2 of type L from entering at m given 1 has entered at m and is of type θ_1. Let $\rho_1 = \frac{\gamma}{\gamma + r_1(1-\gamma)}$ be the interim belief that 1 is of type H where r_1 is the probability with which a type L candidate 1 enters. Then indifference of a type L candidate 2 between entering at m and staying OUT will require

$$\gamma\Pi_2(L|H) + r_1(1-\gamma)\Pi_2(L|L) = 0. \tag{19}$$

Similarly the indifference of candidate 1 of type L yields

$$\gamma\Pi_1(L|H) + r_2(1-\gamma)\Pi_2(L|L) + (1-\gamma)(1-r_2) = 0. \tag{20}$$

Notice that if $r_1 = r_2$ then $\Pi_2(L|H) = \Pi_1(L|H)$ and $\Pi_2(L|L) = \Pi_1(L|L) = 1/2$ so that (20) and (19) cannot hold simultaneously unless $r_2 = 1$. Hence the randomization cannot be symmetric across periods. We now show, as in the proof of Proposition 2 that the voters will strictly prefer candidate 2 to candidate 1 in any two-party contest. First observe that since $(1-\gamma)(1-r_2) > 0$ when $0 < r_2 < 1$, it must be that

$$\gamma\Pi_2(L|H) + r_1(1-\gamma)\Pi_2(L|L) = 0 > \gamma\Pi_1(L|H) + r_2(1-\gamma)\Pi_1(L|L). \tag{21}$$

Next observe that if $r_1 < r_2$ then $\rho_1 > \rho_2$ and hence prior to receiving signals, the voter strictly prefers 1 to 2 when they both stand at m. This means $\Pi_2(L|H) < \Pi_1(L|H)$ and $\Pi_2(L|L) < \Pi_1(L|L)$ so that (21) cannot hold. If $r_1 > r_2$ then $\rho_1 < \rho_2$ and hence prior to receiving signals, the voter strictly prefers 2 to 1 when they both stand at m. This means $\Pi_2(L|H) > \Pi_1(L|H)$ and $\Pi_2(L|L) > \Pi_1(L|L)$ so that (21) holds. It is now easy to see that existence will require conditions on c and signal strength akin to those in Proposition 2.
$\square$

On Pooling equilibria with $n = 2$:

While there cannot be any fully revealing equilibrium, outcomes on the other extreme that deny voters any additional information about the contestants can in principle be possible. Here we show that such pooling equilibria are not possible when both the proportion of high quality parties and the probability of exogenous signals revealing a low quality party are large, even though our dissent equilibria continue to hold. We analyze the case with $n = 2$.

Observation 1. *Unless precision of signals and the cost c are prohibitively high, there exists a unique pooling equilibrium where both candidates compete on a common platform m.*

The intuition behind this goes as follows. Since outcomes generated by any pooling equilibrium cannot affect beliefs held by voters, pooling equilibria must not involve any nontrivial randomization, that is, the support of such a mixture can at most have exactly two ideological positions equidistantly located on either side of m. This is because otherwise any existing entrant will have a strict incentive to deviate and assign all probability on the strictly closest platform in the support of the common mixed strategy. Given this, it is easy to see that if such an equilibrium exists there is policy convergence to the median voter's ideal point m. So suppose there is a pure strategy pooling equilibrium where all types of entrants take the position m. Then each entrant of each type must earn a non-negative expected payoff where expectations are now taken over the common interim belief equal to the prior γ. One can easily show that this expected payoff cannot remain non-negative when the probability of exogenous signals revealing a low quality contestant becomes large and the cost exceeds a certain threshold. At that stage, low quality candidates will strictly prefer to stay OUT.

□

Proof of Proposition 4

Let $Q(z)$ be the probability that a signal is generated for a party with platform $m + z$. Suppose play is according to the strategy profile as in the proof of Proposition 3 and voters hold beliefs therein. Upon observing candidate 1 standing at m, voters believe that any such candidate is of type H with probability γ and a type H candidate 2 earns a payoff equal to 1/2. Consider a deviation by this candidate to z^* where $Q(z) = 1$ for all $z \geq z^*$ and assume that the constituency strictly prefers a candidate at $m + z^*$ to m after exogenous signals are revealed if and only if $s \geq s(z^*;\gamma)$ for some $0 < s(z^*;\gamma) < 1$. Then, the payoff to this deviating candidate 1 is $\mathbb{P}[s \geq s(z^*;\gamma)](1 + c) - c$, and hence this player deviates from the strategy profile if an only if $\mathbb{P}[s \geq s(z^*;\gamma)] > \frac{1+2c}{2(1+c)}$. Since $\frac{1+2c}{2(1+c)} < 1$ for any $c > 0$, the condition is non-empty.

□

Proof of Proposition 5

We will lay out the exact conditions required for this equilibrium. Denote by $\gamma(t)$ the period t prior that a potential entrant is of type H with $\gamma(1) = \gamma$. In this formulation, there are three possible decisions for a candidate: (i) *contest* by committing to a platform, (ii) *wait* for the next round and (iii) *QUIT* the contest forever. For now suppose the only available policies in period t is the set $X_t = \{m - z_t, m, m + z_t\}$ for some $z_t > 0$ and suppose the constituency holds WM beliefs with cut-off z_t for entry in that period. As we will see there are two critical values for these cutoffs. Let h_t be the history at time t that lists the number of current contestants and their positions, H_t be the set of all possible histories at time t and $\mathcal{H}$ the set of all possible histories. Consider the following strategy profile defined for some nonempty set of *terminal* histories $H^* \subset \mathcal{H}$ and a cutoff generating function $\gamma^* : \mathcal{H} \to [0, 1]$ for the prior $\gamma(t)$:

1. If $h_t \in H^*$, then all currently waiting candidates announce QUIT.

2. If for some $h_t \in \mathcal{H} \setminus H^*$ we have $\gamma(h_t) > \gamma^*(h_t)$: all remaining H-types enter at policy $x(h_t) \in \{m - z, m + z\}$ with probability $\alpha(h_t)$ and wait with probability $1 - \alpha(h_t)$ while all L-types wait with probability 1;

3. If for some $h_t \in \mathcal{H} \setminus H^*$ we have $\gamma(h_t) \leq \gamma^*(h_t)$:

   (a) in period t, the remaining H-types enter at policy $x(h_t) \in \{m - z', m + z'\}$ with probability 1 while all L-types do the same with probability $\beta(h_t)$ and wait with probability $1 - \beta(h_t)$;

   (b) in period $t+1$ a fraction $\phi(h_{t+1}) \geq 0$ of the remaining L-type candidates enter at m;

4. The voter updates WM beliefs using Bayes' rule and the exogenous signals unless the pre-signal belief about the type being H across a group of candidates with identical (w.r.t. preferences) policies is 1; in that case the constituency selects that candidate (within this set) who produces the highest signal and then decides whether to vote for him or some other candidate (from outside this set) if available.

Note that H^* is the set of histories such that given any $h_t \in H^*$, c is high enough so that there is no future entry.

'Late' periods:

Suppose we are in a period $t \geq 1$ with history h_t such that $\gamma(h_t) < \gamma^*(h_t)$ and $h_t \notin H^*$. We will write $\#h_t$ to denote the number of existing contestants in the history h_t. According to the above strategy profile, t is therefore the second-last period of the entry game. This also means that all existing contestants in h_t have entered in the set $\{m-z, m+z\}$ and are all believed to be of type H with probability 1 (or no one has entered so far). Further, according to the strategy, all currently waiting type H candidates enter in the set $\{m - z', m + z'\}$ with probability 1. If all L-types are randomizing in period t between entering in the set $\{m - z', m + z'\}$ (with probability $\beta(h_t)$) and waiting (with probability $1 - \beta(h_t)$), at the beginning of period $t + 1$ all remaining candidates reveal themselves as L types. Hence if there is any entry in period $t + 1$, that is, $h_{t+1} \notin H^*$, then each such entry must be at m as prescribed by the strategy profile.

Histories with $\#h_t \geq 1$:

Begin by looking at period t histories with at least one existing contestant who, by virtue of the strategy profile, have been revealed to be of type H. In our construction of the equilibrium, the following property emerges.

Property 1. *For each h_t with $\#h_t \geq 1$ we have $\phi(h_{t+1}) = 0$. It holds whenever $z < h$.*

Property1 means that a H type candidate who has revealed himself by entering before period t has done so with a degree of dissent z such that $z < h$, the utility that a voter receives from the quality parameter of a candidate of type H. Property 1 implies the following:

Fact 2. *If at any history h_t with $\#h_t \geq 1$ we have $\gamma(t) < \gamma^*(h_t)$, then if type L players randomize with probability $\beta(h_t)$ they QUIT with probability $1 - \beta(h_t)$.*

Also note that since the pre-signal beliefs about all entrants in h_t is 1 (since such entries could only be from type H candidates), a type H entrant at t can have a chance of winning, given there is possible mis-representation from L-types, provided the degree of dissent z' in period t is less extreme than z. This gives the next fact.

Fact 3. *In equilibrium it must be that $z' < z$.*

It must also be that each L-type candidate obtains a payoff of 0 from contesting (when their own types contest with probability $\beta(h_t)$) whenever $\#h_t \geq 1$ as in case they do not contest in period t, by Fact 2 they must QUIT (note that such a condition will then guarantee that the H - types who enter in this late period earns a strictly positive payoff in expected terms). Let $\Pi_L(\#h_t, \beta(h_t), \gamma(t))$ be the expected payoff of a type L candidate who enters in period t with dissent z' when all currently waiting candidates enter for sure with dissent z' if of type H and otherwise enter (again with dissent z') with probability $\beta(h_t)$ and QUIT with probability $1 - \beta(h_t)$ if of type L. Define $\gamma^*(h_t)$ as the highest value of $\gamma_t(h_t)$ such that there exists $\beta_t(h_t) \in (0, 1)$ that satisfies the following:

$$\Pi_L(\#h_t, \beta_(h_t), \gamma^*(h_t)) = 0, \#h_t \geq 1. \tag{22}$$

Condition (22) yields the function γ^* for all histories $\#h_t \geq 1$ in which n and c are parameters. Given γ^* solves an indifference condition for L-type candidates, it is clear that H types are entering with probability 1 in period t whenever condition (22) holds. We now explore the condition further. Let $k = (\#h_{t+1} - \#h_t) - 1$ be the number of *other* new entrants in period t when a generic candidate (called i) of type L enters with certainty, $k = 0, \ldots, (n-1) - \#h_t$, when all currently waiting candidates enter for sure with dissent z' if of type H and otherwise enter (again with dissent z') with probability $\beta(h_t)$ and QUIT with probability $1 - \beta(h_t)$ if type L. Let $\mu_t(h_{t+1}|\gamma(h_t))$ be the probability of obtaining h_{t+1} in this situation. Then

$$\begin{aligned}\mu_t(h_{t+1}|\gamma(h_t)) = &\binom{(n-1) - \#h_t}{k} \left(\gamma(h_t)^k (1-\gamma(h_t))^{(n-1)-\#h_t-k}\right) \left((1-\beta(h_t))^{(n-1)-\#h_t-k}\right) \\ &+ (1-\gamma(h_t))^{(n-1)-\#h_t} \binom{(n-1) - \#h_t}{k} \left(\beta(h_t)^k (1-\beta(h_t))^{(n-1)-\#h_t-k}\right).\end{aligned}$$

Let $U_L(\#h_{t+1}|\#h_t, \gamma(h_t))$ be the payoff of this generic type L player i from entering in period t with history h_t when period t actions yield history h_{t+1} for period $t+1$. Then,

$$\Pi_L(\#h_t, \beta_(h_t), \gamma(h_t)) = \sum_{k=0}^{(n-1)-\#h_t} \mu_t(h_{t+1} I|\gamma(h_t)) U_L(\#h_{t+1}|\#h_t, \gamma(h_t)),$$

so that condition 22 becomes

$$\sum_{k=0}^{(n-1)-\#h_t} \mu_t(h_{t+1}|\gamma^*(h_t))U_L(\#h_{t+1}|\#h_t, \gamma^*(h_t)) = 0. \tag{23}$$

Histories with $\#h_t = 0$:

When $\#h_t = 0$ we begin by taking note of the special case when $\#h_{t+1} = 0$, an event that can occur in equilibrium with strictly positive probability. Here the end sub-game is a simultaneous move game played between n candidates of type L. With our assumption of pivotal constituency, all entering at m is the unique Nash equilibrium here with each entrant obtaining a payoff equal to $1/n$, thus, $\phi(\emptyset) = 1$. The following fact summarizes this.

Fact 4. *If* $\gamma(h_t) < \gamma^*(h_t)$, $h_t \notin H^*$ *and* $h_{t+1} = \emptyset$, *then* $\phi(h_{t+1}) = 1$ *with all candidates entering at* m *and earn* $\frac{1}{n}$.

Also, starting from $\#h_t = 0$, the probability of the event h_{t+1} when all (but one) type L players randomize while this player stays OUT (that is, waits) is given by

$$\begin{aligned}\hat{\mu}^-[h_{t+1}|\gamma(h_t)] = \binom{n-1}{\#h_{t+1}}\gamma(h_t)^{\#h_{t+1}}(1-\gamma(h_t))^{(n-1-\#h_{t+1})}(1-\beta(h_t))^{(n-1-\#h_{t+1})} \\ + (1-\gamma(h_t))^{n-1}\binom{n-1}{\#h_{t+1}}\beta(h_t)^{\#h_{t+1}}(1-\beta(h_t))^{(n-1)-\#h_{t+1}}\end{aligned}$$

and when this player enters, this expression becomes

$$\begin{aligned}\hat{\mu}^+[h_{t+1}|\gamma(h_t)] = \binom{n-1}{\#h_{t+1}-1}\gamma(h_t)^{\#h_{t+1}-1}(1-\gamma(h_t))^{(n-1-(\#h_{t+1}-1))}(1-\beta(h_t))^{(n-1-(\#h_{t+1}-1))} \\ + (1-\gamma(h_t))^{n-1}\binom{n-1}{\#h_{t+1}}\beta(h_t)^{\#h_{t+1}}(1-\beta(h_t))^{(n-1)-(\#h_{t+1}-1)}\end{aligned}$$

Let $U_L^+(\#h_{t+1}|\#h_t = 0)$ be the payoff of this generic type L candidate when he enters at t and $U_L^-(\#h_{t+1}|\#h_t = 0)$ when he waits. Then, $U_L^-(\#h_{t+1}|\#h_t = 0)$ is falling in $\#h_{t+1}$ with $U_L^-(0|0) = 1/n$. On the other hand, $U_L^+(\#h_{t+1}|\#h_t = 0)$ is falling in $\#h_{t+1}$ and we (can safely) assume that costs c, signal precision and n are such that with no existing contestants in its history,

1. the expected payoff of a type L candidate from entering today with dissent z' in the event of no one else entering today (and revealing all of them to be of type L) is higher than the payoff from waiting and revealing oneself as type L and entering at m with all other candidates, and
2. the expected payoff from quitting the game when everyone else has entered is higher than entering with the rest.

This is made precise in Assumption 1:

Assumption 1. *$U_L^+(1|0) > 1/n$ and $U_L^-(n-1|0) > U_L^+(n|0)$.*

Let $\Pi_L^+(\#h_t, \beta_t(h_t), \gamma_t(h_t))$ be the expected payoff of an L type candidate who enters at period t with empty history and $\Pi_L^-(\#h_t = 0, n, \beta_t(h_t), \gamma_t(h_t))$ when he stays out. Then,

$$\Pi_L^-(\#h_t, \beta_t(h_t), \gamma_t(h_t)) = \sum_{\#h_{t+1}=0}^{n-1} \hat{\mu}^-[h_{t+1}|\gamma(h_t)] U_L^-(\#h_{t+1}|\#h_t = 0)$$

and

$$\Pi_L^+(\#h_t = 0, \beta_t(h_t), \gamma_t(h_t)) = \sum_{\#h_{t+1}=1}^{n} \hat{\mu}^-[h_{t+1}|\gamma(h_t)] U_L^-(\#h_{t+1}|\#h_t = 0)$$

Since he is randomizing, it must be that

$$\Pi_L^-(\#h_t, \beta_t(h_t), \gamma^*(h_t)) = \Pi_L^+(\#h_t = 0, \beta_t(h_t), \gamma^*(h_t)),$$

yielding

$$\sum_{\#h_{t+1}=0}^{n-1} \hat{\mu}^-[h_{t+1}|\gamma^*(h_t)] U_L^-(\#h_{t+1}|\#h_t = 0) = \sum_{\#h_{t+1}=1}^{n} \hat{\mu}^-[h_{t+1}|\gamma^*(h_t)] U_L^-(\#h_{t+1}|\#h_t = 0). \tag{24}$$

Assumption 1 guarantees that if the entry game arrives at the situation where the current prior about waiting candidates being of type H is equal to γ^*, there exists a randomization between waiting and entering which if used by all other L - type candidates and all H-type candidates on the other hand enter with probability 1, then each L-type candidate is indifferent between these two actions.

'Early' periods:

Pick a generic period $t \geq 1$ with history $h_t \notin H^*$ such that $\gamma_t(h_t) > \gamma^*(h_t)$ where $\gamma^*(h_t)$ is given by (22). According to the above strategy profile, each type H candidate who has not entered is randomizing on dissent z with probability $\alpha_t(h_t)$ while all type L candidates are waiting. Given the tie-breaking rule of 'going with the highest signal' used by voters on a set of candidates with identical dissent where each is believed to be of type H with probability 1, no L types find it beneficial to enter at this stage if c is relatively high and signals are sufficiently informative. So we concentrate only on type H candidates for this phase of the entry game and keep type L candidates waiting until play eventually enters a phase with a future history $h_{t'} \notin H^*$, $t' > t$ and $\gamma(h_{t'}) < \gamma^*(h_{t'})$.

Let H_{t+1}^{+i} be the support of the immediate future history induced by a probability distribution μ^{+i} when a candidate of type H enters and all other type H candidates follow the randomization $\alpha_t(h_t)$. Similarly let H_{t+1}^{-i} be the support of the immediate future history induced by a probability distribution μ^{-i} when this candidate waits. Given a history h_{t+1} where i has not entered, we denote by $h_{t+1} \cup \{i\}$ the history where now i enters with dissent z in that history. The following fact is immediate on the transition probabilities under the strategy profile conditional on wether or not candidate i enters:

Fact 5. *$h_{t+1} \in H_{t+1}^{-i}$ iff $h_{t+1} \cup \{i\} \in H_{t+1}^{+i}$ and $\mu^{-i}(h_{t+1}) = \mu^{+i}(h_{t+1} \cup \{i\})$*

Given h_t, the randomization $\alpha(h_t)$ and the prior $\gamma(h_t)$, let $k = \#h_{t+1} - \#h_t$, $k = 0, \ldots, n - \#h_t$ and let K be the number of type H candidates in the population that have not yet entered, $K = 0, \ldots, n - \#h_t$. It follows that

$$\mu^{+i}(h_{t+1}) = \sum_{K=0}^{(n-1)-\#h_t} \left(\binom{(n-1)-\#h_t}{K} \gamma(h_t)^K (1-\gamma(h_t))^{((n-1)-\#h_t)-K} \right) \left(\binom{K}{k} \alpha(h_t)^k (1-\gamma(h_t))^{K-k} \right)$$

and

$$\mu^{-i}(h_{t+1}) = \sum_{K=0}^{n-\#h_t} \left(\binom{n-\#h_t}{K} \gamma(h_t)^K (1-\gamma(h_t))^{(n-\#h_t)-K} \right) \left(\binom{K}{k} \alpha(h_t)^k (1-\gamma(h_t))^{K-k} \right).$$

For a fixed history h_t define the following subsets of immediate future histories:

$$\hat{H}^+(h_t) := \left\{ h \in \mathcal{H} | h \in H_{t+1}^{+i}, \gamma(h) \le \gamma^*(h) \right\},$$

and

$$\hat{H}^-(h_t) := \left\{ h \in \mathcal{H} | h \in H_{t+1}^{-i}, \gamma(h) \le \gamma^*(h) \right\}.$$

Note that the set of immediate future histories where H players keep randomizing with probability $\alpha(h_{t+1})$ are

$$\tilde{H}^+(h_t) = H_{t+1}^{+i} \setminus \left\{ H^* \cup \hat{H}^+(h_t) \right\},$$

and

$$\tilde{H}^-(h_t) = H_{t+1}^{-i} \setminus \left\{ H^* \cup \hat{H}^-(h_t) \right\}.$$

Recall that $\#h_t$ players are all believed to be type H with probability 1 under the strategy profile. Let $\Pi_H^+(h_t)$ be the expected payoff of a type H player from entering at t with history h_t. Then,

$$\Pi_H^+(h_t) = \sum_{h_{t+1} \in H^*} \mu^{+i}(h_{t+1}) \left(\frac{1-c}{\#h_t} \right) + \sum_{h_{t+1} \in \hat{H}^+(h_t)} \mu^{+i}(h_{t+1}) U_H^+(h_{t+1}) + \sum_{h_{t+1} \in \tilde{H}^+(h_t)} U_H^+(h_{t+1}) \quad (25)$$

where $U_H^+(h_{t+1})$ is the entering candidate's payoff when the game continues outside H^*. Similarly, let $\Pi_H^-(h_t)$ be the expected payoff of a type H player from waiting at t with history h_t and contesting in period $t+1$. Note that delay by one period is necessary and sufficient for our purposes. Then,

$$\Pi_H^-(h_t) = \sum_{h_{t+1}\in H^*} \mu^{+i}(h_{t+1})\cdot 0 + \sum_{h_{t+1}\in \hat{H}^-(h_t)} \mu^{+i}(h_{t+1}) W_H^-(\beta(h_{t+1}),\gamma(h_{t+1}))$$

$$+ \sum_{h_{t+1}\in \tilde{H}^+(h_t)} \mu^{+i}(h_{t+1})\Pi_H^+(h_{t+1})$$

$$= \sum_{h_{t+1}\in \hat{H}^-(h_t)} W_H^-(\beta(h_{t+1}),\gamma(h_{t+1})) + \sum_{h_{t+1}\in \tilde{H}^+(h_t)} \Pi_H^+(h_{t+1}), \tag{26}$$

where $W_H^-(\beta(h_{t+1}),\gamma(h_{t+1}))$ is the payoff of a type H candidate from entering at z' when all other type H candidates do the same while all type L candidates randomize with probability $\beta(h_{t+1})$. In equilibrium we require

$$\Pi_H^+(h_t) = \Pi_H^-(h_t) \text{ for each } t \text{ and } h_t \notin H^* \text{ such that } \gamma_t(h_t) > \gamma^*(h_t). \tag{27}$$

We note that condition 27 ensures type H candidates do not deviate in this stage by waiting with probability 1 while it is never incentive compatible to QUIT at this stage even for a type L player. The proof is completed by showing that during the 'early' stages, the prior $\gamma(h_t)$ falls for each h_{t+1} that is in the support of the strategy profile at history h_t. So note that for any h_t in this range, we have

$$\gamma(h_t) = \frac{(1-\alpha(h_{t-1}))\gamma(h_{t-1})}{(1-\alpha(h_{t-1}))\gamma(h_{t-1}) + (1-\gamma(h_{t-1}))}.$$

We want to show that $\gamma(h_t) < \gamma(h_{t-1})$, that is,

$$\frac{(1-\alpha(h_{t-1}))\gamma(h_{t-1})}{(1-\alpha(h_{t-1}))\gamma(h_{t-1}) + (1-\gamma(h_{t-1}))} < \gamma(h_{t-1})$$

which simplifies to $-\alpha(h_{t-1})\gamma(h_{t-1}) < -\alpha(h_{t-1})\gamma(h_{t-1})^2$ that is always true since $\alpha(h_{t-1}) > 0$. This completes the proof.

□

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